When do proxy advisors improve corporate decisions?

Berno Buechel†, Lydia Mechtenberg‡ and Alexander F. Wagner§

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Abstract

This paper studies the impact of proxy advisors (PAs) on shareholder decision-making. We posit two assumptions: (i) the board of directors has a better signal regarding the value-maximizing decision on a given issue than any single shareholder can have based on own research; (ii) shareholders can condition their investment in information acquisition on the PA’s recommendation. If only assumption (i) holds, shareholders lack research incentives; PAs are not the root cause of suboptimal shareholder incentives. If assumption (ii) additionally holds, shareholders use PA recommendations to identify controversial issues for further investigation. Consequently, a PA improves corporate decision quality.

Keywords: Proxy advisors, strategic voting

JEL classification: G23, D72, D83
1 Introduction

Shareholders vote on a variety of important issues, including director elections, executive compensation, and certain aspects of mergers and acquisitions. During the past two decades, shareholders’ decision making has changed due to the rise of a new business: Proxy advisory firms (such as ISS and Glass Lewis) provide voting recommendations to shareholders. These recommendations have substantial impact on shareholder voting outcomes.\footnote{See, for example, Alexander et al. (2010); Choi, Fisch, and Kahan (2010); Ertimur, Ferri, and Oesch (2013); Iliev and Lowry (2015); Larcker, McCall, and Ormazabal (2013, 2015); Li (2018); Malenko and Shen (2016); McCahery, Sautner, and Starks (2016); Matsusaka and Shu (2021). PAs can also have an impact by increasing shareholder engagement with firms (Dey, Starkweather, and White, 2022).} There is an ongoing public and scientific debate about the effects of proxy advisors (PAs in what follows) on the quality of decision making in shareholder meetings.\footnote{Spatt (2021) provides a recent survey of the literature with a focus on regulatory issues.} The regulation of PAs is highly contentious, as evidenced, for example, by frequent and substantial changes of rules applied by the U.S. Securities and Exchange Commission (SEC) within the last few years (most recently in July 2022, rescinding rules only adopted in 2020).

A key point of contention is that PAs may crowd out shareholders’ incentives to invest in own research. Shareholders who rely on a PA’s recommendation as a substitute for own research save costs individually, but negatively affect the collective by not contributing new information into the decision-making process. This intuition has been probed and developed in the influential analysis of Malenko and Malenko (2019).

In this paper, we show that under two arguably practically relevant assumptions the presence of a PA actually leads to either more shareholders who invest in research or at least not fewer, and hence improves corporate decision quality. The basic intuition is that shareholders who usually, without a PA, “rubber-stamp” any given proposal, use the PA as
a filter to identify those issues that are controversial and hence deserve further investigation.

In our model, there are three types of agents: Shareholders, the firm’s board of directors, and a PA. Shareholders and the board both care about firm value, whereas the PA cares for its profit. There is a proposal on an issue (e.g., a director election). Agents are imperfectly informed about the correct decision on the issue, i.e., about which decision will increase firm value most. The board and the PA receive a private imperfect and independently distributed signal about the correct decision. The board recommends a decision based on its own signal. For brevity, we refer to this as the “board’s proposal.” Then, each shareholder individually decides whether to buy the PA’s vote recommendation, i.e., the PA’s signal, and whether to invest in own research, i.e., to obtain a private signal. Finally, shareholders vote, and the simple majority rule determines the outcome.

Two key assumptions set this model apart from the existing literature. First, we posit that the board is better-informed than any single shareholder alone. This assumption, “BIB” (for better-informed board), is in line with a long tradition of studies in corporate governance arguing that insiders (the board and management) have information about the company that may be superior to that of shareholders (Jensen and Meckling, 1976).

The second key assumption is “PAF” (proxy advice first): after receiving proxy advice, a shareholder can decide upon additional research about the issue at hand. This assumption is more likely to hold in regulatory settings that provide shareholders with sufficient time to conduct research.

We solve our game-theoretic model for pure Perfect Bayesian Nash equilibria, mainly

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3Hence, the PA sells information directly in the sense of Admati and Pfleiderer (1990).
focusing on equilibria that are not Pareto-dominated by other equilibria.\textsuperscript{4} We find that the presence of a PA increases the shareholders’ incentives to invest in own research or leaves these incentives unchanged relative to the case without a PA.

The underlying intuition for this finding is as follows: Begin with the simplest firm, consisting only of one shareholder. Consider how the shareholder thinks about the board’s proposal in the situation \textit{without} a PA. Even if the shareholder has invested into own research and this signal is not in favor of the board’s proposal, she is still better off to vote for the proposal since the board, by Assumption BIB, is better informed than the shareholder. Therefore, the shareholder would not invest into own research in the first place and prefers to follow the proposal.

This simple logic extends to equilibria involving many shareholders. In that case, a given shareholder only needs to consider the case in which her vote decides whether the proposal passes or not, i.e., where the shareholder is pivotal. If she is pivotal, this must mean that the positive and negative signals of other shareholders are equally frequent. In this situation, only her own information and the board’s information are crucial, so we are effectively back in the case with a single shareholder. Thus, none of the shareholders has an incentive to become informed.

While this behavior is efficient in the one-shareholder case, it is typically inefficient when there are multiple shareholders. Substituting the own research with the informativeness of the board’s proposal is individually rational, but collectively harmful as it leads to correlated mistakes. This result holds as long as shareholders do not anticipate the board’s proposal to be so conflicted as to be uninformative, a model variation that we discuss in Section 5.

\textsuperscript{4}Pareto-dominated equilibria are based on coordination failure.
Overall, BIB leads to correlated votes and a lack of own investment incentives, even without any PA. This argument shows that PAs per se are not the root cause of insufficient incentives for shareholders. Compared to this benchmark, the presence of a PA leads to higher decision quality in our model. Intuitively, for a shareholder it pays off to invest in own research when there is sufficient controversy about whether the proposal should be accepted. When the PA’s signal coincides with the board’s signal, then there is not and hence a shareholder prefers to simply accept the uncontroversial issue. By contrast, when the PA’s signal contradicts the board’s, there is sufficient controversy about the issue. Hence, for a shareholder it pays off to invest in an own signal in that situation. Thus, a PA not only contributes an additional signal into the decision-making process, but also triggers shareholders to conditionally generate additional signals. This effect is strongest when the board and the PA are similarly well informed such that their contradicting signals indicate strong controversy.

The beneficial effects of a PA clearly hinge on our two key assumptions, BIB and PAF. If BIB (better-informed board) were violated, the board’s proposal would be so uninformative as to motivate shareholders to invest in their own signals already in the benchmark setting where there is no PA. If PAF (proxy advice first) were violated, a shareholder could no longer condition investing in an own signal on a disagreement between the board’s and the PA’s signals. Both Assumption BIB and PAF appear plausible in practice, and together they yield a simple reason for PAs to exist.

After showing our results for symmetric equilibria, we extend the analysis to asymmetric equilibria. Asymmetric equilibria permit shareholders to specialize on different strategies, even
though shareholders are ex ante identical. In particular, the crucial strategy of conditionally investing in research is still very common in asymmetric equilibria. In addition there are, depending on the parameters, shareholders who always invest in an own signal without subscribing to the PA, shareholders who rubber-stamp the board’s proposals or shareholders who “robo-vote,” i.e., always vote according to the PA’s recommendations. With asymmetric equilibria, too, PAs improve collective decision-making.

Although our model is necessarily stylized, it provides important perspectives on some recent regulatory developments, which we discuss in Section 6. For example, some recent proposals would arguably lead to a failure of Assumption PAF, because timelines around shareholder meetings would be adjusted such that investors simply would not have time to do research contingent on PA advice. Such interventions would, therefore, inadvertently lead to lower quality corporate decision-making.

Our paper contributes to the literature on voting, and in particular to voting in corporations. Seminal works in this literature, elucidating specifically the role of strategic voting, include Maug (1999), Maug and Yilmaz (2002), and Maug and Rydqvist (2009). More broadly, our model relates to the literature on strategic voting in a common interest setting that started with Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996). In particular, our analysis contributes to the study of informational efficiency of such votes. Informational inefficiencies occur when symmetry assumptions of the standard Condorcet model are violated, either with regard to the signal technology (Austen-Smith

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5 Of course, another reason for shareholders to behave heterogeneously in practice is that they differ ex ante. Studies featuring shareholder heterogeneity include Levit, Malenko, and Maug (2021) and Levit, Malenko, and Maug (2022).

6 For studies of informational efficiency in games with strategic complements and strategic substitutes, see Angeletos and Pavan (2007) and Hellwig and Veldkamp (2009).
and Banks, 1996) or with regard to the information-transmission process (Gerardi and Yariv, 2007; Iaryczower, Shi, and Shum, 2018; Buechel and Mechtenberg, 2019). Inefficiencies are also generated when private information becomes costly (Persico, 2004; Gershkov and Szentes, 2009), or when public information is provided that is not of sufficiently high quality (Kawamura and Vlaseros, 2017; Jeong, 2019; Liu, 2019).\footnote{Generally speaking, inefficiencies can be generated by correlation of private beliefs across voters, either through public information or through information-transmission processes between voters that are not optimally tailored to the signal (quality) distribution. Accordingly, Levy and Razin (2015) show that correlation neglect can enhance informational efficiency.} In the corporate finance context, a relatively recent innovation, starting with Malenko and Malenko (2019), has been to enhance the analysis by considering the presence of proxy advisors.\footnote{Other work on (strategic) voting in the corporate finance context includes Brav and Mathews (2011), Levit and Malenko (2011), Matvos and Ostrovsky (2006), Meirowitz and Pi (2022), and Van Wesep (2014), among others.}

We contribute to the literature in two different respects: First, we show that muted shareholder incentives to conduct own research are not solely due to a PA acting as a substitute informer, but also occur \textit{without} a PA if the board’s proposal is based on sufficiently valuable information. Second, we show that proxy advice given early enough can foster shareholders’ own investments in research. This has the policy implication that shareholders need to have sufficient time to conduct such research \textit{after} receiving the advice from PAs. Under these conditions, PAs are likely to improve decision quality and social welfare.

Our model, because of the three-way strategic interaction, is already feature-rich, but it is worth emphasizing that our focus in the analysis is on situations where the goal of the decision-making is to maximize firm value and the PA provides unbiased recommendations. Thus, while we provide a version of the model with conflicts of interest between the board and the shareholders, another conflict of interest, namely, where the PA also sells consultancy
services to the firms and is potentially “captured” by a firm’s management, is not modeled.
We abstract from this problem because we wish to establish when a PA can potentially be value-increasing. This motivation is similar in spirit to Boot, Milbourn, and Schmeits (2006) who establish a reason for credit rating agencies to exist in the absence of conflicts of interest. Also, some studies investigate the idea that biasing recommendations may increase the fraction of shareholders who subscribe to the PA’s service.\(^9\) We see our approach as complementary by establishing under which conditions PAs trigger shareholders’ information acquisition and hence improve corporate decision-making in the absence of these frictions.

The paper proceeds as follows. Section 2 sets up the model. Section 3 provides the main results, which are then illustrated in Section 4. Section 5 shows robustness of the results, in particular with respect to asymmetric equilibria. Regulatory implications are discussed in Section 6. Section 7 briefly discusses potential other applications of our model.\(^{10}\)

2 Model Setup

2.1 Basic Ingredients

We model voting on corporate decisions as strategic voting under uncertainty (Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1996, 1997, 1998). Thus, we follow frameworks such as Malenko and Malenko (2019), Bar-Isaac and Shapiro (2020) and Ma and Xiong

\(^9\)For example, Matsusaka and Shu (2020) show how a PA profitably caters to the preferences of investors. Malenko, Malenko, and Spatt (2021) analyze how a PA can enhance profits by biasing its recommendations against the more likely alternative. In Ma and Xiong (2021), a PA skews its recommendation either because of a conflict of interest or according to a bias on the side of the shareholders. Levit and Tsoy (2020) show how an advisor may adopt one-size-fits-all recommendations in order to obscure its biases.

\(^{10}\)All propositions of the main text are proven in Appendix A in this document. Some further arguments used in the discussion in the main text are formally shown in the Supplementary Online Material (SOM), which is available here: https://bit.ly/proxy-SOM.
A firm is owned by $N > 1$ shareholders, where $N$ is odd. The firm faces uncertainty with respect to a binary decision.\textsuperscript{11} Making the ex post correct decision will increase firm value by an amount normalized to 1, while the wrong decision leaves it unchanged.

More formally, there are two states of the world $\theta \in \{A, B\}$ with equal prior probability. Slightly abusing notation, we assume that the firm has to decide on a binary issue $\{A, B\}$ that yields value 1 if and only if the decision matches the true state.

The board of directors receives a binary signal $s_B$ regarding the issue to be voted on. The signal takes on values $a$ or $b$. The signal quality is $q_B \in (\frac{1}{2}, 1)$, i.e., $Pr[s_B = a|\theta = A] = Pr[s_B = b|\theta = B] = q_B$. Again slightly abusing notation, we assume that the board then recommends either action $A$ or $B$. We call this the “board’s proposal.”

A profit-maximizing proxy advisor (PA) offers advice to shareholders at fee $f > 0$. The PA receives a signal about the true state as well. The quality of that signal is $q_P \in (\frac{1}{2}, 1)$. The PA provides a vote recommendation \textit{for} or \textit{against} the board’s proposal to subscribing shareholders.

Shareholders decide whether to subscribe to the PA’s offer. If a shareholder subscribes, she receives the PA’s recommendation. A shareholder \textit{then} decides whether to invest $c > 0$ in own research about the issue at hand. If a shareholder expends own research costs, this leads to a private signal of quality $q_S \in (\frac{1}{2}, 1)$. When the shareholder meeting is held, each shareholder votes \textit{yes} or \textit{no}. Abstentions are excluded.\textsuperscript{12} For simplicity, each shareholder holds one share of the firm and each share provides one vote. The decision that receives a

\textsuperscript{11}Examples vary by jurisdiction and include but are not restricted to director elections, dividends, shareholder proposals, compensation-related matters, etc.

\textsuperscript{12}Practically, shareholders may also abstain. However, according to most institutional settings abstentions are counted (either as \textit{yes} or \textit{no}) and hence shareholders’ voting action is essentially binary.
majority of votes is implemented. Conditional on state $\theta$, all signals are independent, and precision levels $q_B$, $q_P$, and $q_S$ are common knowledge.

Our first leading assumption is that the board knows better than any single shareholder what is good for the company.

**Assumption 1 (BIB).** The board is at least as well informed as a single shareholder, i.e.,

$$q_S \leq q_B.$$  

“BIB” stands for better-informed board. For the quality of the PA $q_P$ we do not make an assumption that restricts it to be above or below the other agents’ qualities.

In the course of the analysis it will come in handy to transform signal qualities $q \in (0, 1)$ into log-odds $\log(\frac{q}{1-q}) \in (0, \infty)$. We denote the log-odds of the board being correct as $\ell_B := \log(\frac{q_B}{1-q_B})$ and likewise $\ell_S := \log(\frac{q_S}{1-q_S})$ for the shareholders and $\ell_P := \log(\frac{q_P}{1-q_P})$ for the PA. Then Assumption BIB reads $\ell_S \leq \ell_B$.\textsuperscript{13} This notation is convenient since it allows us to aggregate signal qualities by summation. To see this, consider the board’s signal $b$ and assume, for instance, that both the PA and one shareholder have received signals $a$ and that there is no further information. Then, the board’s signal is rather correct than not if and only if $q_B(1 - q_P)(1 - q_S) \geq (1 - q_B)q_Pq_S$, which is equivalent to $\ell_B \geq \ell_P + \ell_S$.

Our second leading assumption is that shareholders can condition their research investment on the PA’s recommendation.

**Assumption 2 (PAF).** Subscribing shareholders decide upon own research investment after

\textsuperscript{13}Nitzan and Paroush (1982) show that among voters with idiosyncratic signal precision the optimal voting weights would be according to these log-odds.
they have received the PA’s recommendation.

“PAF” stands for “proxy advice first”. Shareholders may conduct a bulk of their general research about a company independent of the proxy advice and also before receiving the PA’s recommendation. Our assumption PAF is that the information relevant for deciding on a specific issue can be conditioned on the PA’s recommendation.  

2.2 Simplification and Timeline

It turns out that we can substantially simplify the exposition without losing substance of the analysis by fixing the signal and behavior of the board and the behavior of the PA. The board receives a signal and then makes a recommendation. We let the board’s signal be always b (for board). We fix the board’s behavior by assuming it makes the proposal according to its signal, i.e., it has received signal b and now proposes action B. Likewise, we fix the PA’s behavior to set fee f > 0 and recommend according to its signal, i.e., it recommends for if it has received signal b (for board) and it recommends against if it has received signal a (against board).

These simplifications do not affect the substance of our analysis and results. Since the board, like the shareholders, aims at maximizing firm value, revealing its signal to help shareholders decide for the optimal policy is in its own interest. The proxy advisor, in turn, can generate profits only from helping individual shareholders to make a decision that is

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14Relaxing this assumption would change the timing of our model such that shareholders have to decide simultaneously about subscribing to the PA and about investing in own research. That is the assumption in Malenko and Malenko (2019). We show the consequences of making this assumption in our model in Supplementary Online Material (SOM) Section 1.3 and discuss it in the main text.

15This will exclude strategies that depend on the label of the alternative, such as always voting yes for alternative A and no for alternative B independent of which alternative the board has proposed.

16In Section 5, we discuss to which extent, and with which implications, a re-interpretation of our model covers conflicts of interest between board and shareholders.
even more informed than it would be without the PA. The reason is that the shareholders’
willingness to pay for proxy advice depends only on the PA’s contribution to the informedness
of the vote.\footnote{In Malenko, Malenko and Spatt (2022), the PA biases a costless public recommendation to enhance the shareholders’ perceived pivotality since their willingness to pay for the PA’s costly signal increases in pivotality. Heterogeneity in voters, which allows these manipulations to be effective, is created by subjective values.} Hence, the optimal strategy of the PA is to reveal its true signal whenever
asked to do so, and to set the fee $f$ equal to the shareholder’s willingness to pay for this
revelation. If the willingness to pay is negative, the PA will be driven out of the market. In
our setting, the PA stimulates rather than crowds out own research for any given fee that
allows the PA to be in the market. Thus, the PA’s profit maximization does not conflict
with efficiency. Hence, explicitly including a strategic PA and an endogenous fee does not
add any interesting results in our setting. These insights are captured by our simplifying
assumptions, to focus the analysis on the main implications of the model.

The timeline, which is illustrated by Figure 1, summarizes the simplified setup. At $t = 0$
nature draws a state of the world and signals for all potential recipients of signals. At $t = 1$
each shareholder decides whether to pay the fee for the PA’s report. Those who pay the fee
receive the truthful vote recommendation which is equivalent to learning the PA’s signal. At
$t = 2$ each shareholder decides whether to invest costs $c$ to receive an own independently and
identically distributed signal of quality $q_S$. At $t = 3$ shareholders vote. At $t = 4$ the proposal
passes if a majority approves it and payoffs are realized.

### 2.3 Strategies

The most important strategic aspects concern the shareholders. They have several strategies
both on the information acquisition stages ($t = 1$ and $t = 2$, respectively) and on the
Each shareholder can subscribe to vote recommendation to learn signal of PA
Each shareholder can invest into research to receive own signal
Each shareholder casts vote
Majority decision implemented and payoffs realized

Figure 1: Timeline. For simplicity, the board’s and PA’s behavior is fixed. In particular, the PA’s recommendation strategy is fixed to be truthful such that subscribing shareholders learn the PA’s signal. (Actions in italics only apply if there is a PA.)

voting stage \((t = 3)\). On the information acquisition stages, there are six strategies: A shareholder who does not subscribe may invest in own research \(\text{NotSubscribe-Invest}\) or not \(\text{NotSubscribe-NotInvest}\); a shareholder who does subscribe may unconditionally invest in research \(\text{Subscribe-Invest}\) or not \(\text{Subscribe-NotInvest}\) or, else, may invest in research only if the recommendation is \(\text{for}\) \(\text{Subscribe-InvestIFF for}\) or only if the recommendation is \(\text{against}\) \(\text{Subscribe-InvestIFF against}\).

In the voting stage, any shareholder chooses \text{yes} or \text{no}. The set of voting strategies depends on the acquired information which may include the PA’s signal and the own signal. For instance, for a shareholder who acquired both kinds of information (e.g., with \text{Subscribe-Invest}), a voting strategy is a mapping \(v_i : \{\text{for, against}\} \times \{a, b\} \to \{\text{yes, no}\}\). Slightly abusing notation, we write \(\sigma_i\) for the information acquisition and voting strategy of a shareholder \(i\), and we use \(\sigma = (\sigma_1, ..., \sigma_N)\) to denote a strategy profile of shareholders.

We study Perfect Bayesian Nash equilibria, i.e., players best respond to their beliefs and update their beliefs according to Bayes’ rule wherever possible. We focus on pure strategies, but analyze both symmetric and asymmetric strategy profiles. Mixed equilibria are often interpreted as pure-strategy equilibria of different players, that is, players do not literally...
randomize between the strategies, but the probability weight on each pure strategy rather represents the fraction of the population that plays it. Hence, they are similar to asymmetric equilibria in pure strategies, which capture heterogeneous behavior of shareholders more directly. Therefore, we do not additionally treat mixed equilibria. When there are multiple equilibria in some area, we exclude those that are Pareto-dominated by other equilibria. This eliminates equilibria that are due to miscoordination.

To analyze the model we take the perspective of a regulator who compares a market with a PA, as in the game defined above, with a market in which no PA is admitted. The regulator maximizes welfare which coincides with maximizing decision quality in our setup. We will assume that costs of information acquisition, be it fee $f$ or costs $c$, are relatively small compared to benefits on decision quality. Hence, when shareholders have to trade off costs of information acquisition with benefits of higher firm value, we will assume that the latter dominates. When there are two strategies with the same decision quality, then shareholders strictly prefer the one with lower costs, as we assume that costs are strictly positive. The quality of corporate decisions is measured by $\Pi(\sigma)$, the ex ante probability that the decision will match the true state.\footnote{This is also called \textit{informational efficiency}, which can be distinguished from \textit{economic efficiency} (see, e.g., Buechel and Mechtenberg, 2019). Economic efficiency means welfare, which here can be defined as $\Pi(\sigma)$ net of the investment costs in own research since the prices paid to the PA are transfers. When investment costs $c$ become arbitrarily small, the two concepts coincide.} In what follows, all proofs are relegated to Appendix A.

3 Main Results

**Benchmark Setting without Proxy Advisor.** Consider first the benchmark case that no PA is admitted. Thus, posit that in the timeline of Figure 1 actions at $t = 1$ are suppressed.
Then a shareholder’s information acquisition decision reduces to whether to acquire an own signal or not in $t = 2$. Suppose for a moment that all shareholders do acquire such a signal and vote according to it. We call this strategy profile UNIS, for “UNconditional Investment in own Signal,” where the term “unconditional” will be justified later, when shareholders could potentially condition their investment in own research on the PA’s vote recommendation.\footnote{For simplicity, we use the same labels for strategies as for the symmetric strategy profiles composed of these strategies, e.g., we speak of UNIS both to denote the strategy to invest in an own signal and to vote according to it and to denote the strategy profile in which all shareholders do so. The precise meaning of these labels will be obvious from the context.} In this strategy profile the decision quality amounts to $\Pi(\sigma^{UNIS}) = \pi(N)$, where $\pi(N) := \sum_{i=\frac{N+1}{2}}^{N} \binom{N}{i} q_i^s (1 - q_S)^{N-i}$ is the probability that a majority decision of $N$ shareholders is correct.

While the decision quality of such voting behavior is usually very high (De Caritat, 1785), it is unfortunately not an equilibrium under Assumption BIB. The intuition is straightforward once spelled out. A single shareholder $i$ can improve by deviating to not acquire a signal and vote $yes$. When this shareholder $i$ is pivotal, the signals of all $N - 1$ other shareholders are split: there are as many $a$-signals as there are $b$-signals among them. Now, even if $i$’s signal points against the board’s proposal, Assumption BIB, i.e., the assumption that the board is at least as well informed as $i$, makes it beneficial to vote $yes$, i.e., for the board’s proposal, and not to acquire own information in the first place. We call this latter strategy and its corresponding strategy profile “Rubber-stamping”.\footnote{Assumption BIB, $q_S \leq q_B$, is in fact necessary and sufficient for Proposition 1. Since we have Assumption BIB as a leading assumption, we only show sufficiency in the proof of Proposition 1.}

**Proposition 1** (SYM without PA). *Suppose no PA is admitted. If Assumption BIB holds, then there does not exist a symmetric equilibrium in which shareholders invest in own research. Hence, decision quality in symmetric equilibria is bounded by: $\Pi(\sigma) \leq q_B$. The*
Pareto-efficient\footnote{Recall that the criterion of Pareto-efficiency is applied within the set of symmetric equilibria. Within the set of all strategy profiles, there might well be strategies that Pareto-dominate the Pareto-efficient symmetric equilibrium.} symmetric equilibrium is Rubber-stamping and leads to decision quality

\[ \Pi(\sigma^{Rubber}) = q_B. \]

Proposition 1 shows that without a PA the quality of decision making is bounded by the quality of the board whose proposal is rubber-stamped by the shareholders. This result is similar to the substitution effect of Malenko and Malenko (2019), but occurs on a different level: For the scenario that the board’s proposals are uninformative, Malenko and Malenko (2019) obtain over-reliance of shareholders on the PA’s recommendations; for the scenario that the board’s proposals are informative, we obtain over-reliance on this proposal without any PA, as a new benchmark.\footnote{The result that no shareholder invests in own research is stylized. When studying asymmetric equilibria without PA, we find the same message in a less stylized form: Without PA, there always exist shareholders who do not invest in own research, while the number of shareholders who do is bounded. For symmetric mixed-strategy equilibria, similar results to our analysis of asymmetric pure-strategy equilibria, as discussed in Section 5.1, are expected.}

Including Proxy Advice. The presence of a PA substantially increases a shareholder’s set of information-acquisition strategies. One of them, Subscribe-InvestIFF\textit{against}, gives rise to the following symmetric strategy profile, which we denote by \( \hat{\sigma} \) and call “\textit{CAIS} (Conditional on Advice Invest in Signal)”: All shareholders subscribe to proxy advice; if the recommendation is \textit{for}, they vote \textit{yes}; if the recommendation is \textit{against}, they invest in own research and vote according to their own signal, i.e., vote \textit{yes} if the signal is \( b \) and \textit{no} if it is \( a \).

In this strategy profile shareholders use the PA’s recommendation as a filter: \textit{for} recommendations are followed without being challenged; \textit{against} recommendations trigger further
investigation of the issue. CAIS is illustrated in Table 1.

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<thead>
<tr>
<th>PA’s recommendation</th>
<th>Own Signal</th>
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<tbody>
<tr>
<td>for b (for board)</td>
<td>a (against board)</td>
</tr>
<tr>
<td>against yes</td>
<td>no</td>
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<td>no</td>
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</tbody>
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Table 1: Strategy CAIS: Invest in research if and only if vote recommendation is against; and after for recommendation vote yes, after against recommendation vote yes if and only if signal is for board.

It turns out that based on this strategy profile the negative result of Proposition 1 can be mitigated by the presence of a PA, as Proposition 2 shows.

**Proposition 2** (SYM with PA). Let Assumptions BIB and PAF hold. Let costs \( c \) be arbitrarily small and let fee \( f \) be sufficiently smaller. Suppose there is a PA with \( \ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S) \). Then there exists a symmetric equilibrium in which shareholders conditionally invest in own research. The Pareto-efficient equilibrium is CAIS and leads to decision quality \( \Pi(\hat{\sigma}) > q_B \). Otherwise (i.e., if \( \ell_P \notin (\ell_B - \ell_S, \ell_B + \ell_S) \)), the Pareto-efficient equilibrium is Rubber-stamping with \( \Pi(\sigma_{Rubber}) = q_B \).

In the proof of Proposition 2 (Appendix A.1) we proceed as in the proof of Proposition 1 (Appendix A.2), by providing first the full characterization of all equilibria as a lemma and then selecting those that are not Pareto-dominated. Comparing Proposition 2 with Proposition 1, we conclude that the presence of a PA either strictly improves decision quality or leaves it unchanged, compared to the setting without a PA. The condition for the strict improvement can be rewritten as \( |\ell_B - \ell_P| < \ell_S \), which has the following interpretation: the difference in quality of board and PA is smaller than the information quality of one shareholder. If this conditions is satisfied, there is no equilibrium with information acquisition.
without a PA, while we have a new equilibrium (CAIS) with a PA in which all shareholders conditionally invest in own research.

The first intuition for the conditions of the PA being beneficial as stated in Proposition 2 can be seen from their violations. Consider the symmetric strategy profile CAIS. If \( \ell_P \leq \ell_B - \ell_S \), we have \( \ell_S + \ell_P \leq \ell_B \), i.e., the board is better informed than the PA and one shareholder together. Then there is a deviation from CAIS to Rubber-stamping. Intuitively, the board is sufficiently well informed that it does not individually pay off to acquire any information, even if it were costless. If \( \ell_P \geq \ell_B + \ell_S \), i.e., the PA is better informed than the board and one shareholder together, then there is a deviation from CAIS to not investing and to voting against the board’s proposal. Indeed, the deviating shareholder’s vote is only pivotal if board and PA disagree and voting no improves decision quality, given that the PA is so well informed. If costs \( c \) or \( f \) are not small enough, there is again a beneficial deviation, e.g., to Rubber-stamping, which saves costs. Finally, if the PA’s fee \( f \) is not sufficiently smaller than the costs \( c \), then deviating to UNIS saves costs without affecting the outcome.\(^{23}\) In Section 1 of the Supplementary Online Material (SOM) we extend the analysis to the complete characterization of all symmetric equilibria in pure strategies, with and without Assumptions BIB and PAF. Most importantly, that analysis shows that the two key assumptions Assumption BIB and PAF are not only sufficient but also necessary for the conclusion, as we will discuss below.

\(^{23}\)The assumption \( c \) small enough assures that shareholders who can improve decision quality by investing in own research would not shy away due to the high costs. The assumption that the costs are larger than zero matters when deviations that do not affect decision quality are considered. The assumption that fees \( f \) are sufficiently smaller than \( c \) means that the results answer the question whether there is a fee \( f \) such that a PA can profitably be active in the market.
4 Illustration and Discussion

Numerical Example. To get a better understanding of Propositions 1 and 2, consider Example 1.

Example 1 (Symmetric Equilibria). Let $q_B = 0.75$, $q_P = 0.7$, and $q_S = 0.6$. Then $\ell_B = 0.477$, $\ell_P = 0.368$, and $\ell_S = 0.176$ such that the condition $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$ of Proposition 2 is satisfied, as $0.368 \in (0.477 - 0.176, 0.477 + 0.176)$. Table 2 illustrates the implications of Propositions 1 and 2 for decision quality. First, not admitting a PA leads to Rubber-stamping and hence to a decision quality of $q_B = 0.75$, independent of the number of shareholders $N$ (Proposition 1). Second, when a PA is admitted, CAIS is the Pareto-efficient symmetric equilibrium, which delivers a strictly higher decision quality (by Proposition 2). Its decision quality is further increasing in the number of shareholders $N$ and approaching 0.925 < 1 for large $N$. Finally, Table 2 shows the hypothetical case in which all shareholders play UNIS, i.e., invest in own research. This is not an equilibrium but a classic benchmark capturing the quality of majority decisions by $N$ sincere voters, as already pointed out by the Marquis de Condorcet (De Caritat, 1785). In this benchmark case, decision quality may start low, but becomes larger than in equilibrium for a sufficiently large number of voters.

<table>
<thead>
<tr>
<th>Setting</th>
<th>Decision quality</th>
<th>$N = 3$</th>
<th>$N = 5$</th>
<th>$N = 21$</th>
<th>$N = 101$</th>
<th>$N = 1,001$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No PA</td>
<td>$\Pi(\sigma^{\text{Rubber}}) = q_B$</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>With PA</td>
<td>$\Pi(\hat{\sigma}) = q_Bq_P + p^{\text{dis}}\pi(N)$</td>
<td>0.784</td>
<td>0.798</td>
<td>0.855</td>
<td>0.917</td>
<td>0.925</td>
</tr>
<tr>
<td>Hypothetical</td>
<td>$\Pi(\sigma^{\text{UNIS}}) = \pi(N)$</td>
<td>0.648</td>
<td>0.683</td>
<td>0.826</td>
<td>0.979</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 2: Decision quality in Example 1. The table considers the two Pareto-efficient symmetric equilibria, Rubber-stamping and CAIS, and strategy profile UNIS, which is not an equilibrium. Illustration of Propositions 1 and 2 for $q_B = 0.75$, $q_P = 0.7$, and $q_S = 0.6$. $p^{\text{dis}} := (1 - q_B)q_P + q_B(1 - q_P)$ is the probability that the board’s and the PA’s signal differ.

We now turn to illustrating Propositions 1 and 2 graphically, while at the same time...
extending our analysis to the entire parameter space.

**Graphical Illustration.** Figure 2 illustrates the full parameter space, including the areas where Assumption BIB is violated. An entry \((x, y)\) in this coordinate system has the simple interpretation that the board is equally well informed as \(x\) shareholders, while the PA is equally well informed as \(y\) shareholders.\(^{24}\)

In the upper panel of Figure 2, no PA is admitted. By Proposition 1, Rubber-stamping is the Pareto-efficient equilibrium under Assumption BIB, i.e., \(q_S \leq q_B\). This is illustrated in the area \(\ell_P/\ell_S \geq 1\). Assumption BIB is necessary and sufficient for this conclusion as UNIS is the Pareto-efficient equilibrium for \(\ell_P/\ell_S < 1\). Hence, when there is no PA, information acquisition occurs if and only if the board is less well informed than a single shareholder, i.e., when Assumption BIB is violated.\(^{25}\)

In the lower panel of Figure 2, there is a PA and Assumption PAF is satisfied. Proposition 2 has shown that under Assumption BIB, i.e., for \(\ell_P/\ell_S \geq 1\), we have either CAIS or Rubber-stamping as Pareto-efficient symmetric equilibrium. Moreover, by Proposition 2 the parameter space in which CAIS is an equilibrium is given by the condition \(\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)\), which defines a corridor around the 45-degree line.\(^{26}\) On the 45-degree line the board and the PA are exactly equally well informed, i.e., \(\ell_P/\ell_S = \ell_B/\ell_S\) (or \(q_B = q_P\)). Note that this corridor is not bounded from the upper right. Hence, for arbitrarily well-informed board and PA, there is still an equilibrium with conditional information acquisition of all shareholders, as long

\(^{24}\)”Equally well informed” means here that if \(x\) shareholders have received a signal \(a\) (against the board) then both states \(A\) and \(B\) are equally likely. Hence, if more than \(x\) shareholders have received a signal \(a\) and there is no other information, then the board should be overruled.

\(^{25}\)The additional results, due to the violation of Assumption BIB, are provided in the Supplementary Online Material (SOM) Section 1.

\(^{26}\)When studying asymmetric equilibria, we show that CAIS can be played by a majority of shareholders far beyond this corridor. The corridor only restricts the area in which all shareholders play CAIS.
as the board and the PA are roughly equally-well informed. The intuition is that whenever their signals contradict each other, there is sufficient controversy to invest in own research.

To further understand the workings of the model, consider the comparative statics of changing information quality. Assume $\ell_P > 1$ and start with an uninformed PA: $q_P \approx 0.5$ i.e., $\frac{\ell_P}{\ell_S} \approx 0$. Decision quality remains unaffected by the PA’s information quality $q_P$ (or $\frac{\ell_P}{\ell_S}$) at first, then discontinuously increases from $q_B$ to $\Pi(\hat{\sigma})$. Within the region where CAIS is an equilibrium, decision quality further improves as $\Pi(\hat{\sigma})$ is continuously increasing in $q_P$. Finally, it returns to the level $q_B$ when Rubber-stamping is played again. Hence, there is a non-monotonic effect of a PA’s information quality on the corporate decision quality with the latter being highest for a PA that is slightly better informed than the board.\(^{27}\) Comparative-static effects of the board’s information quality are analogous if $\ell_P > 1$, i.e., the PA is better informed than a single shareholder. Finally, increasing signal quality of the shareholders, $q_S$, reduces $\ell_B$ and $\frac{\ell_P}{\ell_S}$, which means graphically moving towards the origin. This improves decision quality of CAIS as shareholders base their decision on their own information when the PA’s recommendation is against.

Assumption BIB, i.e., $\frac{\ell_B}{\ell_S} \geq 1$, rules out UNIS, the strategy profile in which all shareholders acquire information. Violating BIB while satisfying PAF, UNIS is the Pareto-efficient symmetric equilibrium in the lower left corner of the parameter space (in the lower panel of Figure 2), which is defined by the condition $\ell_S > \ell_B + \ell_P$. Hence, in the presence of a PA, UNIS requires that one single shareholder must be better informed than board and PA

\(^{27}\)A non-monotonic effect of the PA’s recommendation quality on the corporate decision quality is also predicted by the analysis of Malenko and Malenko (2019). In their model, higher PA quality always weakly reduces the shareholders’ investment in private signals such that maximal research incentives are obtained for the lowest PA quality. In our model, maximal research incentives are obtained for intermediate PA quality, namely when it equals the board’s quality.
together. Interestingly, this is an even stronger condition than the condition for UNIS when no PA is admitted ($\ell_S > \ell_B$).

![Parameter space with Pareto-efficient symmetric equilibria. Upper panel: without a PA; lower panel: with a PA. Both panels: The areas to the left of $\ell_B = 1$ are precluded by Assumption BIB; they are still depicted here to show the effect of the assumption.]

**Figure 2**: Parameter space with Pareto-efficient symmetric equilibria. Upper panel: without a PA; lower panel: with a PA. Both panels: The areas to the left of $\ell_B = 1$ are precluded by Assumption BIB; they are still depicted here to show the effect of the assumption.

Let us now compare the upper panel with the lower panel. Under Assumption BIB, i.e., for $\frac{\ell_B}{\ell_S} \geq 1$, the presence of a PA weakly improves decision quality, as it replaces Rubber-stamping with CAIS if anything. When Assumption BIB is violated, there can be a different effect. Suppose that the quality of the board is not much better than a coin flip, i.e., $q_B \approx 0.5$. Then $\frac{\ell_B}{\ell_S} \approx 0$ and there is the equilibrium with full information acquisition (UNIS) and high decision quality, as long as no PA is admitted. The presence of a PA who is better informed than a single shareholder ($\frac{\ell_P}{\ell_S} > 1$) destroys this equilibrium and reduces decision quality.
from $\pi(N)$ to $q_B \approx 0.5$. The reason is that conditional on pivotality a shareholder prefers to follow the PA’s recommendation over acquiring and using the own signal. This is similar to the substitution effect already established for PAs (Malenko and Malenko, 2019). Hence, Assumption BIB dramatically changes how admission of a PA affects decision quality when studying symmetric equilibria.

5 Extensions and Robustness

5.1 Asymmetric Equilibria

The main text characterizes the (Pareto-efficient) symmetric equilibria. We can drop the symmetry assumption. Since the characterization of all (Pareto-efficient) asymmetric equilibria is quite cumbersome and involves many case distinctions, we relegate it to the Supplementary Online Material (SOM) Section 2 and provide only the main result and the essence of the other findings here. Interestingly, although we model shareholders as ex ante homogeneous, there is specialization on different strategies in the Pareto-efficient equilibria, e.g., in one typical equilibrium, some shareholders play CAIS, some shareholders play UNIS, while others play either Rubber-stamping or always follow the PA’s recommendation, depending on whether the board or the PA is better informed.

Our basic results remain similar. In particular, we can first show that without PA, the number of shareholders who invest in own research is bounded from above. That is, in the equilibrium without a PA there are always some shareholders not investing in research, given that Assumption BIB is satisfied. Second, when admitting a PA whose signal quality is not
too far from the board’s, the number of shareholders who invest or conditionally invest weakly increases. Again, the basic idea is that the PA’s recommendation is used as a condition to invest in own research like in information-acquisition strategy Subscribe-InvestIFF against, which constitutes CAIS. While this was true for all shareholders in Proposition 2 in a certain parameter range, we now find this in much larger areas of the parameter space, while no longer all shareholders use this strategy. More precisely, if we have $\frac{|\ell_B-\ell_P|}{\ell_S} < \frac{N+1}{2}$, then $N - \frac{|\ell_B-\ell_P|}{\ell_S}$, i.e., more than half of all shareholders, invest into own information, either conditionally as in CAIS or even unconditionally as in UNIS. The above condition means that the difference between the information quality of the PA and the information quality of the board must not exceed the aggregated information quality of about half of all shareholders together, which graphically widens the corridor in the lower panel of Figure 2 from starting at 1 to starting at $\frac{N+1}{2}$ on both axes.\(^{28}\) Moreover, the number of investing shareholders, $N - \frac{|\ell_B-\ell_P|}{\ell_S}$, is decreasing in this difference of information quality. Hence, as in the analysis of symmetric equilibria, we find the strongest research incentives for shareholders when the PA is as well informed as the board.\(^{29}\)

Finally, the question remains how the effects of a PA on equilibrium behavior translates into decision quality. Proposition 3 provides the answer.

**Proposition 3 (ASYM).** Let costs $c$ and $f$ be sufficiently small. For any setting of signal qualities $q_B, q_P, q_S \in (\frac{1}{2}, 1)$, decision quality in any Pareto-efficient equilibrium with a PA under Assumption PAF, $\Pi(\sigma^*)$, is weakly higher than decision quality in any strategy profile without a PA (including their Pareto-efficient equilibria), i.e., $\Pi(\sigma^*) \geq \bar{\Pi}_{no-PA}$, where

\(^{28}\)Observe that the larger the number of shareholders $N$, the less demanding this assumption is.

\(^{29}\)Other comparative-static effects might be different for asymmetric equilibria than for symmetric, see Supplementary Online Material (SOM) Section 2.
\(\Pi^{\text{no-PA}}\) is the maximal decision quality for any strategy profile in the game without a PA.

The proof of Proposition 3 considers all Pareto-efficient strategy profiles and shows that each of them is an equilibrium with maximal decision quality.\(^{30}\) It then uses the insight that any decision quality without a PA can be replicated with a PA who is ignored. As a consequence, decision quality cannot be reduced.\(^{31}\) In sum, the novel type of equilibrium behavior that we find in this paper exists in a broad range of the parameter space. The main insight, that PAs weakly improve decision quality, holds even for the whole parameter space when considering asymmetric strategy profiles.

### 5.2 One Dominant Shareholder

We have thus far assumed that \(N > 1\) and odd which means that we have at least three shareholders. Let us now consider the case of only one shareholder \(N = 1\), which applies to any company with a shareholder who holds a decisive majority of shares. We can show that both main results carry over to this case. First, without a PA, there is no incentive to invest in research under Assumption BIB, i.e., for \(q_S \leq q_B\). Second, the presence of a PA with appropriate information quality improves decision quality, as it leads to a Pareto-efficient equilibrium in which the shareholder conditionally invests in research.

Interestingly, since one single shareholder is always pivotal, the Assumption PAF is not necessary for research investment in that special case. That is, even when the subscription decision and the information acquisition decision are made simultaneously, there is an equilibrium with investment in own research for \(N = 1\). In this equilibrium strategy the

\(^{30}\)Indeed, for asymmetric equilibria there is no issue of inefficiency.

\(^{31}\)This simple line of argumentation does not apply to symmetric equilibria.
shareholder subscribes to the vote recommendation and invests in own research (Subscribe-Invest) and votes yes if and only if at least one of the two supports the board’s proposal. Hence, for the case of only one shareholder, there is a complementarity between proxy advice and own research, independently of the timing of the two decisions.

5.3 Conflicts of Interest

In a re-interpretation of our model the board has a partial conflict of interest with the shareholders. Suppose the effect of conflicted interests is that this reduces the likelihood that the board’s proposal is correct from $q_B > 0.5$ to some $\tilde{q}_B > 0.5$. More technically, suppose the board’s bias is a random variable that is drawn and private information. Shareholders know the distribution of the bias, but not its realization. The distribution of the bias is such that either the board’s proposal is determined by the bias or that it is determined by the signal. Moreover, suppose that the board’s bias is symmetrically distributed around zero.\footnote{Asymmetry of the bias distribution would make one type of proposal more informative than the other.} This introduces noise into the informativeness of the board’s proposal as shareholders put positive probability on the case that the proposal is independent of the signal. Then the assumption of a high quality board $\tilde{q}_B \geq q_S$ thus means that the board is not only better informed, but also that the board’s agency problem is limited. Conversely, a low $\tilde{q}_B$ means either that the board has a low signal quality or that it has a high agency problem such that its proposal is not very informative.

Reconsidering the comparative-statics on $q_B$ (cf., e.g., Figure 2), we can thus also address how the agency problem affects the decision quality. Start with a very well informed board and a small agency problem: $q_B > \tilde{q}_B > q_P > q_S$. Reducing $\tilde{q}_B$ first fosters the shareholders’
research incentives up to the point $q_B \approx q_P$, then reduces them. This non-monotonicity makes it possible that agency problems may even increase the quality of corporate decision making, as boards whose proposals are less informative may incentivize shareholders to (conditionally) invest in own research.

6 Practical and Regulatory Implications

6.1 Shareholder Behavior in Practice

Our analysis aligns well with empirical observations on shareholder behavior. In practice, PAs deliver, together with the vote recommendation, comprehensive materials providing background and detailing the reasoning, especially for negative recommendations. When shareholders review these materials, this is a form of own shareholder research (in addition to research they might conduct separately). A survey containing responses of asset managers including 24% of world-wide assets under management found that 70% of international asset managers (almost all of whom purchase proxy advisory services) see value in receiving these additional materials (Swipra, 2018). Hence, our theoretical insight that the PA provides the investor with valuable optionality to acquire information is in line with evidence on how many investors use PAs in reality.

While many large investors thus at least occasionally collect information in addition to the voting recommendation, some investors in practice seem to simply rubber-stamp (the board’s proposal) or to engage in “robo-voting” (the PA’s recommendation). For example, Shu (2021) documents that in 2017, about a quarter of ISS customers and a small fraction of
Glass Lewis customers have simply followed the PA. These behaviors are covered by our model, in two ways. First, shareholders in our model who conditionally invest in own research might appear to have rubber-stamped or robo-voted, as they in fact very often vote in line with the board or the PA (always after *for*, frequently after *against* recommendations). Second, when we extend the analysis to asymmetric equilibria, there are some shareholders who really find it optimal to always vote with the board (without investing in own research or subscribing to the proxy advisor) or to always buy and follow the PA’s recommendation.

In our analysis robo-voting is never pre-dominant (as it only appears in the analysis of asymmetric equilibria, in a part of the parameter space where the PA is better informed than the board and it is never played by a majority of shareholders). We consider this as realistic. If a majority of equally-sized shareholders robo-voted, a negative vote recommendation would always lead to rejection of the proposal. Empirical evidence (see references in footnote 1), however, shows that, while negative PA recommendations do have negative effects on the share of *yes* votes, most proposals endorsed by the board still pass in practice by a wide margin. This observation hence rather suggests that there are shareholders who conditionally invest in own research or rubber-stamp, the predominant behavior in our (symmetric and asymmetric) equilibria. In other words, while over-reliance on the PAs recommendation is an already known diagnosis, our model diagnoses that shareholders may over-rely on the informativeness of the board’s proposal.

In the Supplementary Online Material (SOM) Section 2, this is embodied in SOM Lemma 2.3 and illustrated SOM Figure 2.1., where the robo-voting strategy is called *Follow.*
6.2 Regulation of Timing of Proxy Advice

To discuss recent policy developments related to the timing of proxy advice, we first summarize how a different timeline would affect our results. Consider the situation when proxy advice arrives after the shareholders’ decision to invest in own research, i.e., when Assumption PAF is violated. All actions occur as illustrated in the timeline (Figure 1), but proxy advice arrives at the end of period $t = 2$. We consider the cases where Assumption BIB holds and where it does not hold. If Assumption BIB holds, the Pareto-efficient symmetric equilibrium is Rubber-stamping and hence decision quality is bounded by $q_B$. Hence, there is no positive effect of having a PA, as decision quality with or without a PA is bounded by the quality of the board.

If Assumption BIB does not hold, that is, if the board does not have the best information regarding what is good for the company, we find that UNIS is an equilibrium and Pareto-efficient if and only if $\ell_S \geq \ell_B + \ell_P$; otherwise, Rubber-stamping is the Pareto-efficient equilibrium. This condition is the same as in our model with early proxy advice (see bottom right corner of the lower panel of Figure 2). It is more demanding than the condition in the setting without a PA, which was $\ell_S > \ell_B$. Specifically, the condition $\ell_S \geq \ell_B + \ell_P$ means that a single shareholder has to be better informed, not only than the board, but than both the board and the PA together. Hence, the introduction of a PA whose information arrives late, if anything, weakens the shareholders’ research incentives.

In sum, introducing a PA whose advice does not arrive sufficiently early does not induce equilibria with higher decision quality, but may even reduce decision quality. The positive effects of proxy advice in our model are hence indeed restricted to having both Assumption BIB
and Assumption PAF satisfied.\textsuperscript{34}

These insights are important in light of recent policy developments. In August 2019, the U.S. Securities and Exchange Commission (SEC) issued guidance for investment advisors, stating that investment advisors satisfy their own fiduciary duties of care and loyalty and obligations to act in their clients’ best interests in part through careful oversight of proxy advisory firms. Such oversight involves monitoring and analyzing the methodology and processes of proxy advisory firms, including their processes for engagement with companies and procedures to address errors.\textsuperscript{35} In other words, blindly following the vote recommendation is seen to violate an investment advisor’s fiduciary duties to its clients. The rule implies that indeed investors need to have (and take) enough time to conduct their own research.

A later proposed SEC rule from November 2019 instead would presumably have left little time for shareholders to do independent research after receiving the proxy advisors’ recommendations.\textsuperscript{36} In particular, under that rule, companies would have typically three to five days to respond to the initial recommendation of the proxy advisor, so that proxy advisor clients would receive recommendations only later. This would in turn substantially shorten the time available to investors. (For example, there are on average 13 trading days between the date ISS issues its voting recommendation and the meeting date (Li, Maug, and Schwartz-Ziv, 2022).) The rule adopted in July 2020, however, addressed this issue to some extent.\textsuperscript{37} It stated that PAs have to provide their clients with a mechanism by which they can reasonably be expected to become aware of any written statements regarding the voting advice by the companies that are the subject of that advice in a timely manner before

\textsuperscript{34}The exemption is $N = 1$ when a PA can improve decision quality even if PAF is violated; see Section 5.2.

\textsuperscript{35}\url{https://www.sec.gov/rules/interp/2019/ia-5325.pdf}

\textsuperscript{36}\url{https://www.sec.gov/rules/proposed/2019/34-87457.pdf}

\textsuperscript{37}\url{https://www.sec.gov/rules/final/2020/34-89372.pdf}
the shareholder meeting. In fact, in November 2021, the SEC, under a new chairman, went even further in a new proposal.\textsuperscript{38} Under the new rule, PAs would now not be required to engage with the companies that are the subjects of their advice. SEC Chairman Gary Gensler motivated this rule by saying that proxy advisor clients “deserve to receive independent proxy voting advice in a timely manner.”\textsuperscript{39} This rule was adopted in July 2022.\textsuperscript{40}

The requirement for investors to have enough time for own research is in line with the model’s prediction that in such a case high decision-quality can arise. However, we also note that in the novel equilibrium behavior that we find (CAIS), for recommendations are directly accepted, while only against recommendations trigger further research. It remains to be seen whether such partial own research fulfills the fiduciary duties in the eyes of the SEC.

\section*{6.3 Regulation of Proxy Advisor Competence}

In parallel with issuing the guidance for investment advisors discussed above, in August 2019, the SEC also issued guidance and interpretation on the role of PAs.\textsuperscript{41} The guidance includes recommendations on disclosure of the sources of information and methodology used by PAs and information regarding conflicts of interests. Similarly, the EU has also adopted disclosure rules for PAs in the new EU Shareholder Rights Directive (Directive (EU) 2017/828 amending Directive 2007/36/EC).\textsuperscript{42} Such disclosure regimes may support the enhancement

\footnotesize{\textsuperscript{38}https://www.sec.gov/rules/proposed/2021/34-93595.pdf
\textsuperscript{40}https://www.sec.gov/rules/final/2022/34-95266.pdf
\textsuperscript{41}https://www.sec.gov/rules/interp/2019/34-86721.pdf
\textsuperscript{42}PAs are required to publish a code of conduct which they apply and to report the application of the code (or explain why they do not have a code or deviate from it). Member states shall require PAs to publicly disclose certain information, such as, the main features of a PA’s methodology, the main information sources used and the procedures put in place to ensure the quality of research, advice and voting recommendation. Finally, member states must ensure that PAs identify and disclose actual or potential conflicts to their clients. Disclosure itself is limited to the client, i.e., the institutional investors.}

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of proxy advisor competence through market discipline.

While the August 2019 SEC guidance focuses on disclosure requirements, the July 2020 rule mentioned above presumably more directly enhanced pressure on PAs to produce high-quality reports. Specifically, as noted above, that rule would have required PAs to provide their clients with a mechanism by which they can reasonably be expected to become aware of any written statements regarding its proxy voting advice from the subject companies, in a timely manner before the shareholder meeting. Moreover, the SEC reaffirmed that it considers PAs to be under SEC regulation governing solicitation and also defined instances where omitting information in a PA report could constitute fraud.\textsuperscript{43} Under the above-mentioned new rule from July 2022, in order to be exempt from the proxy rules, PAs need to comply only with the disclosure requirements regarding conflicts of interest. Besides not requiring proxy advisors to engage with the companies that are the subjects of their advice, the SEC also removed the examples of situations in which the failure to disclose certain information in proxy voting advice may be considered misleading.\textsuperscript{44}

Arguably, the July 2020 rules would have implied a higher signal quality for the PA. In our model, this implies the following effect: A better informed PA up to a certain level (namely, the level of the information quality of the board) encourages information acquisition by the shareholders and improves decision quality. This holds if proxy advice is early enough for shareholders to condition their research investment on it. If instead proxy advice does not arrive sufficiently early or if the PA is already better informed than the board, a competence-increasing regulation of the PA may undermine shareholders’ research incentives.

\textsuperscript{43}The purpose was not to get PAs to file proxy statements; rather, by fulfilling certain actions, they would be granted an exemption from doing so, an indirect way of regulating their activities.

\textsuperscript{44}See Cooley (2022) for a non-technical summary of the background and the most recent ruling.
and affect decision quality negatively. Importantly, our results show that PAs need not be better informed than the board to improve corporate decision quality.

7 Conclusion

In this paper we analyze the effects of proxy advisory firms (PAs) on corporate decisions. As a benchmark PAs are not admitted. When the board’s proposals are sufficiently informative, shareholders do not have incentives to conduct their own research and simply rubber-stamp the board’s proposals. Hence in the absence of PAs, decision quality is bounded by the quality of the board (Proposition 1). Introducing a PA whose information level is not too far from the board’s alters this result and leads to a higher decision quality (Proposition 2). This only holds if the vote recommendation of the PA arrives sufficiently early such that shareholders can respond to against recommendations with an own investigation of the issue. Extending the analysis from symmetric equilibria to asymmetric equilibria, we find that many but not all shareholders play this conditional investment strategy. Importantly, we arrive at the same overall conclusion: PAs improve corporate decision quality (Proposition 3).

While our model is motivated by the world of shareholder meetings, other practically relevant situations in principle have similar features. For example, Sangiorgi and Spatt (2017) argue that credit rating agencies can “crowd out” independent information production by investors. Future work might analyze under which conditions credit rating agencies positively contribute to information production.

Even more broadly, in many situations, committee members or other team members making majority decisions are faced with the question of whether to acquire information
in addition to what the chairman or group leader proposes, or whether to rubber-stamp proposals put in front of them. Thus, our model provides insights into whether having a separate advisor’s signal available to all committee and team members could improve decision-making, or whether it would undermine individual incentives to become informed.
A Appendix: Proofs

A.1 Proof of Proposition 1

To prove Proposition 1 Lemma A.1 is helpful.

Lemma A.1 (SYM without PA: All Equilibria). Let Assumption BIB hold. Suppose no PA is admitted.

i. Define Protest as the strategy profile in which no shareholder invests in research and all shareholders vote no. Protest is a symmetric equilibrium for any \( q_B, q_S \in (0.5, 1) \). Its decision quality is \( 1 - q_B \).

ii. Rubber-stamping (i.e., no shareholder invests in research and all shareholders vote yes) is a symmetric equilibrium for any \( q_B, q_S \in (0.5, 1) \). Its decision quality is \( q_B \).

iii. There are no other symmetric equilibria.

Proof. We address each part separately.

i. We have \( N \geq 3 \) shareholders (because \( N > 1 \) and odd). When all shareholders vote no, a single shareholder is never pivotal. Hence, there is no way to increase decision quality. Deviations can thus only affect costs. Since no information is acquired in this information-acquisition strategy (NotSubscribe-NotInvest), costs are minimal. Hence, there is no unilateral improvement.

Decisions always implement the opposite of the board’s proposal. By assumption of the simplified model, the board’s proposal corresponds to its signal (\( B \)). Hence, the ex ante probability that the true state matches the decision equals the probability that the board’s signal does not match the true state, which is \( 1 - q_B \).

ii. The proof that Rubber-stamping is an equilibrium is fully analogous to part i. of Lemma A.1. With Rubber-stamping, the decision quality equals the ex ante probability that the board’s signal matches the true state, which is \( q_B \).

iii. There are only two information-acquisition strategies. For not investing in an own signal both strategies are symmetric equilibria (see part i. and ii.). Consider now investment in an own signal: Since shareholders pay \( c \) they must condition on their own signal. Otherwise, they could improve their utility by voting the same and not investing \( c \). Conditioning on their signal leaves two pure strategies: vote yes if \( b \) and no if \( a \) (i.e., UNIS) or the opposite (vote yes if \( a \) and no if \( b \)). If voting yes after \( a \) (against) was optimal, then voting no after \( a \) would also be so. Hence, shareholders could profitably deviate to unconditionally voting \( A \).

We finally show that UNIS is not an equilibrium under Assumption BIB, i.e., \( q_S \leq q_B \).\textsuperscript{45}

\textsuperscript{45}In fact, UNIS is a symmetric equilibrium if and only if \( q_S > q_B \).
only if \( i \) is pivotal and the own signal is \( a \): Under UNIS \( i \) would vote \( \text{no} \), under Rubber-stamping \( i \) would vote \( \text{yes} \). Pivotality implies that among the \( N - 1 \) other shareholders the signals are split in \( \frac{N-1}{2} \) \( a \)-signals and \( \frac{N-1}{2} \) \( b \)-signals. Conditional on that case, \( B \) is more likely to be true than \( A \) (such that Rubber-stamping weakly improves decision quality) if and only if

\[
q_B (1 - q_S) \left( \frac{N - 1}{N-1} \right)^{\frac{N-1}{2}} (1 - q_S) \frac{N-1}{2} \geq (1 - q_B) q_S \left( \frac{N - 1}{N-1} \right) (1 - q_S) \frac{N-1}{2} q_S^{\frac{N-1}{2}}
\]

\[
q_B (1 - q_S) \geq (1 - q_B) q_S
\]

\[
\frac{q_B}{1 - q_B} \geq \frac{q_S}{1 - q_S}
\]

\[
\ell_B \geq \ell_S.
\]

Hence, Rubber-stamping weakly improves decision quality for \( q_S \leq q_B \), which is Assumption BIB. Moreover, Rubber-stamping saves costs \( c \). Therefore, it strictly improves utility of the deviating shareholder \( i \).

Now, we use Lemma A.1 to prove Proposition 1. Under Assumption BIB there are only two equilibria. Equilibrium Rubber-stamping leads to the same costs as the Protest equilibrium. Rubber-stamping Pareto-dominates Protest because it leads to higher decision quality \( \Pi(\sigma^{\text{Rubber}}) = q_B > 0.5 > 1 - q_B = \Pi(\sigma^{\text{Protest}}) \).

### A.2 Proof of Proposition 2

To prove Proposition 2 Lemma A.2 is helpful.

**Lemma A.2** (SYM with PA: All Equilibria). Let Assumptions BIB and PAF hold. Let costs \( c \) be arbitrarily small and let fee \( f \) be sufficiently smaller.

i. Protest (i.e., no shareholder invests in research and all shareholders vote \( \text{no} \)) is a symmetric equilibrium for any \( \ell_B, \ell_S \in (0, \infty) \). Its decision quality is \( 1 - q_B \).

ii. Rubber-stamping (i.e., no shareholder invests in research and all shareholders vote \( \text{yes} \)) is a symmetric equilibrium for any \( \ell_B, \ell_S \in (0, \infty) \). Its decision quality is \( q_B \).

iii. CAIS is a symmetric equilibrium if and only if \( \ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S) \). Its decision quality is: \( \Pi(\bar{\sigma}) = q_B q_P + [(1 - q_B) q_P + q_B (1 - q_P)] \pi(N) \), with \( \pi(N) := \sum_{i=1}^{N-1} \binom{N}{i} q_S^i (1 - q_S)^{N-i} \).

iv. CAIS-2 is a symmetric equilibrium if and only if \( \ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S) \). Its decision quality is: \( \Pi(\sigma^{\text{CAIS-2}}) = (1 - q_B)(1 - q_P) + [(1 - q_B) q_P + q_B (1 - q_P)] \pi(N) \).

v. There are no other symmetric equilibria. In particular, there is no equilibrium in which all shareholders subscribe to proxy advice and unconditionally invest in own signal (Subscribe-Invest).

**Proof.** We address each part of Lemma A.2 separately.
i. The proof is identical to the proof of Lemma A.1, part i.

ii. The proof is identical to the proof of Lemma A.1, part ii.

iii. CAIS is illustrated in Table 1. We show that CAIS is an equilibrium if and only if $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$.

Suppose first that $\ell_P \notin (\ell_B - \ell_S, \ell_B + \ell_S)$, i.e., either $\ell_P \leq \ell_B - \ell_S$ or $\ell_P \geq \ell_B + \ell_S$. We show that CAIS cannot be an equilibrium. In CAIS pivotality implies that the vote recommendation is against and that among the $N - 1$ other shareholders the signals are split in $\frac{N-1}{2}$ $a$-signals and $\frac{N-1}{2}$ $b$-signals. (Indeed, after recommendation for no shareholder is pivotal.)

Let $\ell_P \leq \ell_B - \ell_S$. Consider a shareholder $i$ who deviates to Rubber-stamping. This deviation alters the decision in comparison to CAIS if the vote recommendation is against, all other shareholder’s signals are split, and $i$’s signal is $a$: In CAIS, $i$ would vote no, in the deviation $i$ would vote yes. This deviation weakly improves decision quality if $\ell_B \geq \ell_P + \ell_S$, which holds by assumption. Since, the deviation saves costs $c$, it increases $i$’s expected utility.

Let $\ell_P \geq \ell_B + \ell_S$. Consider a shareholder $i$ who deviates to voting no without information acquisition (as in Protest). This deviation alters the decision in comparison to CAIS if the vote recommendation is against, all other shareholders’ signals are split, and $i$’s signal is $b$: In CAIS, $i$ would vote yes, in the deviation $i$ would vote no. This deviation weakly improves decision quality if $\ell_P \geq \ell_B + \ell_S$, which holds by assumption. Since the deviation saves costs $c$, it increases $i$’s expected utility.

Hence, if $\ell_P \notin (\ell_B - \ell_S, \ell_B + \ell_S)$, CAIS is not an equilibrium.

Now suppose that $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$. In order to show that CAIS is an equilibrium, we demonstrate that there is no individual deviation that improves utility. We use the following principle: if a deviation is more attractive than another deviation in terms of utility, then excluding the former is sufficient to exclude the latter. We organize the potential deviations by information-acquisition strategy. There are six information-acquisition strategies to consider. Pivotality always implies that the vote recommendation is against and that among the $N - 1$ other shareholders the signals are split in $\frac{N-1}{2}$ $a$-signals and $\frac{N-1}{2}$ $b$-signals.

1. NotSubscribe-NotInvest. Deviating to NotSubscribe-NotInvest and voting yes as in Rubber-stamping is not an improvement for low enough costs given $\ell_S + \ell_P > \ell_B$. This deviation only changes the outcome if the PA has recommended against, $i$ has received signal $a$ (against), and all other shareholders’ signals are split. It would
weakenly improve decision quality iff
\[
q_B(1 - q_P)(1 - q_S) \left( \frac{N - 1}{N - 1} \right) q_S^{\frac{N - 1}{2}} (1 - q_S)^{\frac{N - 1}{2}} \geq (1 - q_B)q_Pq_S \left( \frac{N - 1}{N - 1} \right) (1 - q_S) \frac{N - 1}{2} q_S^{\frac{N - 1}{2}}
\]
\[
q_B(1 - q_P)(1 - q_S) \geq (1 - q_B)q_Pq_S
\]
\[
\frac{q_B}{1 - q_B} \geq \frac{q_P}{1 - q_P} + \frac{q_S}{1 - q_S}
\]
\[
\ell_B \geq \ell_P + \ell_S.
\]

By assumption \( \ell_P > \ell_B - \ell_S \), this deviation strictly decreases decision quality. It does save costs \( f \) always and \( c \) with probability \( q_B(1 - q_P) + (1 - q_B)q_P \). For low enough costs \( f \) and \( c \), Rubber-stamping does not increase utility because of its lower decision quality.

Deviation to vote \( no \) without information acquisition (as in Protest) is not an improvement for low enough costs given \( \ell_P < \ell_B + \ell_S \).

(2) NotSubscribe-Invest. Deviation NotSubscribe-Invest and voting according to the own signal as in UNIS does not change the outcome. Indeed, after a \( for \) recommendation \( i \) is not pivotal, after an \( against \) recommendation \( i \) votes under her deviation as she does under CAIS. Hence, this deviation is an improvement only if it saves costs. It is not an improvement if \( f \leq c[q_Bq_P + (1 - q_B)(1 - q_P)] \), which is satisfied if \( f \) is sufficiently lower than \( c \).

(3) Subscribe-NotInvest. The deviation to buying the PA’s recommendation and following it is not an improvement given \( \ell_P < \ell_B + \ell_S \) and low enough \( c \).

(4) Subscribe-Invest. Deviation to buy both recommendation and signal. Case 1, illustrated in Table A.2, is outcome equivalent, but more costly. Case 2, illustrated in Table A.3, is not an improvement given \( \ell_P < \ell_B + \ell_S \).

(5) Subscribe-InvestIFF \( \text{for} \). Consider the deviation to buying the PA’s recommendation and investing in an own signal iff the recommendation is \( \text{for} \). The case illustrated in Table A.4 is not an improvement given \( \ell_P < \ell_B + \ell_S \). The alternative case, which differs by voting \( yes \) after the \( against \) recommendation, is not an improvement given \( \ell_S + \ell_P > \ell_B \).

(6) Subscribe-InvestIFF \( \text{against} \). Consider the deviation to applying the same information-acquisition strategy as in CAIS, but a different voting strategy. The most attractive deviation is to vote \( no \) after the \( \text{for} \) recommendation. This is outcome equivalent and equally costly and, hence, not an improvement.

Hence, under the conditions assumed in part iii. of the Lemma CAIS is an equilibrium.

Finally, concerning decision quality, notice that if board and PA receive the same signal, this signal determines the decision, and if they receive a different signal, the signal that is received by a majority of shareholders determines the decision. Therefore, decision quality in CAIS is \( (q_Bq_P) \ast 1 + (1 - q_B)(1 - q_P) \ast 0 + [(1 - q_B)q_P + q_S(1 - q_P)]\pi(N) \ast 1, \) as \( q_Bq_P \) is the probability that the board and the PA both receive the same and correct
signal, and \([(1 - q_B)q_P + q_B(1 - q_P)]\) is the probability that the two receive signals that are different from each other.

iv. CAIS-2 is illustrated in Table A.1.

<table>
<thead>
<tr>
<th>PA</th>
<th>Own Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(b) (for board)</td>
</tr>
<tr>
<td>for</td>
<td>no</td>
</tr>
<tr>
<td>against</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table A.1: CAIS-2: Invest in research iff vote recommendation is against; after for recommendation vote no, after against recommendation vote yes iff the own signal is \(b\).

The proof that CAIS-2 is an equilibrium if and only if \(\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)\) is identical to the proof that CAIS is an equilibrium under these conditions (cf. Proof of Lemma A.2, part iii.).

Concerning decision quality, notice that if board and proxy advisor receive the same signal, the decision is contrary to this signal, and if they receive different signals, the signal that is received by a majority of shareholders determines the decision. Therefore, decision quality in CAIS-2 is \((q_Bq_P) \times 0 + (1 - q_B)(1 - q_P) \times 1 + [(1 - q_B)q_P + q_B(1 - q_P)]\pi(N) \times 1\).

v. To show that there are no additional equilibria, we exhaustively discuss all pure strategies. Again, we organize the discussion by information-acquisition strategy.

(1) NotSubscribe-NotInvest. There are only voting strategies yes or no. Both lead to equilibria as shown in parts i and ii.

(2) NotSubscribe-Invest. Since shareholders pay \(c\) they must condition on their own signal. Otherwise, they could improve their utility by voting the same and not investing \(c\). Conditioning on the own signal leaves two pure strategies: vote yes if the signal is \(b\) and no if the signal is \(a\) (i.e., as in UNIS) or the opposite (vote yes if the signal is \(a\) and no if the signal is \(b\)). If voting yes after \(a\) (against) was optimal, then voting no after \(a\) would also be optimal. Hence, shareholders could improve their utility by unconditionally voting \(A\). Only UNIS remains. Under Assumption BIB, NotSubscribe-NotInvest and voting yes as in Rubber-stamping is a profitable deviation from UNIS.

(3) Subscribe-NotInvest. Since shareholders pay \(f\) they must condition on the PA’s recommendation. For instance, they vote yes after for and no after against; or they do the opposite. In either case, no shareholder is pivotal since all vote for the same, given a particular recommendation.

A shareholder can improve her utility by not paying \(f\) and voting, e.g., yes. Hence, there is no symmetric equilibrium with this information-acquisition strategy.

---

46This is not surprising, as both strategies have the same information-acquisition strategy, Subscribe-InvestIFF Against, and they only differ in a voting action, where no player is pivotal.
Subscribe-Invest. Since shareholders pay both $f$ and $c$ they must condition their voting strategy on both the PA’s vote recommendation and the own signal. Otherwise, they could improve their utility with the same voting behavior, but saving costs. This means that in fact only two voting strategies remain.

Case 1: vote \textit{yes} except if the PA’s recommendation is \textit{against} and the own signal is \textit{a}, as in Table A.2. In this case no shareholder is pivotal if the PA recommends \textit{for}. Hence, shareholder $i$ can only be pivotal if the recommendation is \textit{against}. If so, $i$ would vote according to her signal. Hence, deviating to unconditionally investing in an own signal and voting accordingly, as in UNIS, would not change the outcome because either $i$ is not pivotal or $i$ would also vote according to the signal. However, acting as in UNIS saves fee $f$. Thus, this is a profitable deviation, and the strategy profile of case 1, illustrated in Table A.2, cannot be an equilibrium.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{Own Signal} & \textit{b} (for board) & \textit{a} (against) \\
\hline
\textit{for} & yes & yes \\
\textit{against} & yes & no \\
\hline
\end{tabular}
\caption{A strategy profile based on acquiring both proxy advice and own signal, case 1: Subscribe-Invest and vote \textit{yes}, except if PA’s recommendation is \textit{against} and the own signal is \textit{a}.}
\end{table}

Case 2: vote \textit{no} except if both the PA’s recommendation is \textit{for} and the own signal is \textit{b}, as in Table A.3. The analogous argument as above for case 1 applies, as follows: In this case no shareholder is pivotal if the PA recommends \textit{against}. Hence, shareholder $i$ can only be pivotal if the recommendation is \textit{for}. If so, $i$ would vote according to signal. Hence, deviating to unconditionally investing in an own signal as in UNIS would not change the outcome because either $i$ is not pivotal or $i$ would also vote according to the signal. Acting as in UNIS, however, saves fee $f$. Thus, strategy profile of case 2 cannot be an equilibrium.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{Own Signal} & \textit{b} (for board) & \textit{a} (against) \\
\hline
\textit{for} & yes & no \\
\textit{against} & no & no \\
\hline
\end{tabular}
\caption{A strategy profile based on acquiring both proxy advice and own signal, case 2: Subscribe-Invest and vote \textit{no}, except if PA’s recommendation is \textit{for} and the own signal is \textit{b}.}
\end{table}

Therefore, there cannot be a symmetric equilibrium with this information-acquisition strategy (Subscribe-Invest), in which shareholders unconditionally buy both the PA’s recommendation and an own signal.

Subscribe-InvestIFF\textit{for}. Since shareholders pay $f$ and sometimes $c$ they must condition their voting strategy on the recommendation and the own signal when
they acquire them. In particular, after having bought the own signal on top of the recommendation \textit{for}, shareholders must vote according to their signal in equilibrium. Voting the opposite is dominated, and not conditioning as well. This leaves two cases, which we address as Cand. 5a and Cand. 5b. We show that none of them is an equilibrium under Assumption 1. \footnote{In fact, each of these strategy profiles is a symmetric equilibrium if and only if \( \ell_S > \ell_B + \ell_P \).} Consider first Cand. 5a: shareholders vote \textit{yes} except if the PA’s vote recommendation is \textit{for} and the own signal is \textit{a} (against) as in Table A.4.

<table>
<thead>
<tr>
<th>PA</th>
<th>Own Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>\textit{b} (for board)</td>
</tr>
<tr>
<td>\textit{for}</td>
<td>yes</td>
</tr>
<tr>
<td>\textit{against}</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table A.4: Cand. 5a. A strategy profile based on acquiring an own signal iff the recommendation is \textit{for}: Subscribe-InvestIFF \textit{for} and vote \textit{yes}, except if PA’s recommendation is \textit{for} and the own signal is \textit{b}.

Consider shareholder \( i \) who deviates to NotSubscribe-NotInvest and voting \textit{yes}, as in Rubber-stamping. This deviation only alters the outcome when the vote recommendation is \textit{for}, all other shareholders’ signals are split, and \( i \)’s signal is \textit{a} (\textit{against}): Under Cand. 5a, \( i \) would vote \textit{no}, but under her deviation she votes \textit{yes}. Decision quality improves by this deviation if \( \ell_B + \ell_P > \ell_S \). This condition is satisfied by Assumption 1. Moreover, the costs are lower under this deviation than under Cand. 5a. Hence, Cand. 5a cannot be an equilibrium.

Now consider Cand. 5b. Shareholders vote \textit{no} except if the PA’s vote recommendation is \textit{for} and the own signal is \textit{b}. Again, no shareholder is pivotal after recommendation \textit{against}. Hence, deviating to NotSubscribe-NotInvest and voting \textit{yes} as in Rubber-stamping is an improvement, identical to the case of Cand. 5a above.

(6) Subscribe-InvestIFF \textit{against}. Since shareholders pay \( f \) and sometimes \( c \) they must condition their voting strategy on the recommendation and the own signal when they acquire them. In particular, after having bought the own signal on top of the recommendation \textit{against}, shareholders must vote according to their signal in equilibrium. Voting the opposite is dominated, and not conditioning as well. This leaves two cases: \textit{CAIS} and \textit{CAIS-2}, which we have addressed. Hence, there are no further equilibria.

Now we can turn to the proof of Proposition 2. Suppose there is a PA with \( \ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S) \). To show that \textit{CAIS} is an equilibrium and Pareto-efficient, we use Lemma A.2, which shows that besides \textit{CAIS} there are three further equilibria in this paremeter space: Rubber-stamping, Protest, and \textit{CAIS-2}. It remains to show that \textit{CAIS} Pareto-dominates in this area.
First, CAIS has the same costs as CAIS-2 and decision qualities are: $\Pi(\hat{\sigma}) = q_B q_P + [(1 - q_B) q_P + q_B (1 - q_P)] \pi(N)$. $\Pi(\sigma^{CAIS-2}) = (1 - q_B) (1 - q_P) + [(1 - q_B) q_P + q_B (1 - q_P)] \pi(N)$. CAIS has higher decision quality iff $q_B q_P > (1 - q_B) (1 - q_P)$, which always holds as $q_B, q_P > 0.5$. Hence, CAIS Pareto-dominates CAIS-2.

Second, decision quality of Rubber-stamping is $q_B$ and decision quality of Protest is $1 - q_B < q_B$. CAIS has strictly higher decision quality than both iff

\[
\frac{\pi(N)}{1 - \pi(N)} \cdot \frac{q_P}{1 - q_p} > \frac{q_B}{1 - q_B}
\]

\[
\log \left( \frac{\pi(N)}{1 - \pi(N)} \right) + \log \left( \frac{q_P}{1 - q_p} \right) > \log \left( \frac{q_B}{1 - q_B} \right)
\]

\[
\ell_N + \ell_P > \ell_B, \tag{A.1}
\]

where $\ell_N := \log \left( \frac{\pi(N)}{1 - \pi(N)} \right)$.

Since $\ell_N > \ell_S$ and by assumption $\ell_P > \ell_B - \ell_S$, we have $\ell_N + \ell_P > \ell_S + \ell_P > \ell_B$. Hence, CAIS leads to strictly higher decision quality than both Rubber-stamping and Protest. It induces higher costs $f$ and $c$. Thus, for low enough costs, CAIS Pareto-dominates them.

Now, suppose that $\ell_P \not\in (\ell_B - \ell_S, \ell_B + \ell_S)$. To show that the Pareto-efficient equilibrium is Rubber-stamping, we use again Lemma A.2. Under Assumption BIB and for $\ell_P \not\in (\ell_B - \ell_S, \ell_B + \ell_S)$, only two equilibria remain: Rubber-stamping and Protest. Rubber-stamping Pareto-dominates because it leads to higher decision quality $\Pi(\sigma^{Rubber}) = q_B > 0.5 > 1 - q_B = \Pi(\sigma^{Protest})$, while it induces the same costs.

### A.3 Proof of Proposition 3

Suppose there is a PA and Assumption PAF holds.\(^{48}\) Let $S$ be the set of all pure strategy profiles.\(^{49}\) Let $\Pi : S \to [0, 1]$ be the decision quality. Let $S^{max} \subset S$ be the set of all strategy profiles that maximize $\Pi$. As $S$ is finite, $S^{max}$ is non-empty.

A player’s strategy is called minimal if the voting strategy conditions on all pieces of information that are acquired. For instance, consider information-acquisition strategy Subscribe-InvestIFFagainst: When an own signal has been acquired after vote recommendation again, the voting behavior must differ between signal realization $a$ and signal realization $b$ to be part of a minimal strategy. Observe that for any strategy that is not minimal, the voting behavior can be mimicked by a strategy with lower costs, saving $c$ or

\(^{48}\)We are particularly thankful to Maximilian Janisch and Thomas Leh´ericy who suggested this proof idea for this proposition. Interestingly, it can be applied to asymmetric equilibria, but not to symmetric equilibria. The reason is that when studying symmetric equilibria, the space under consideration changes because deviations to strategies that thus form an asymmetric strategy profile are admitted.

\(^{49}\)Then there are 16 strategies for each shareholder.
A strategy profile is called minimal if all players’ strategies are minimal and if any player’s reduction of information acquisition (not subscribing and/or not acquiring an own signal) changes the outcome with positive probability. Now let $S^* \subset S^{\text{max}}$ be the strategy profiles that are minimal and lead to maximal decision quality.

We first show, as Claim 1, that all $\sigma^* \in S^*$ are equilibria. A shareholder can only achieve higher utility than in $\sigma^*$ by higher decision quality or lower costs. Higher decision quality is impossible per definition. Lower costs reduce decision quality because $\sigma^*$ is minimal and decision quality is maximal.

Second, we show, as Claim 2, that any Pareto-efficient strategy profile must belong to $S^*$. Suppose first that $\sigma'$ is Pareto-efficient, but not in $S^*$. Then it is either not maximizing decision quality or not minimal. If it does not maximize decision quality, take another strategy profile, say $\sigma^*$, that does and every shareholder is better off. The reason is that any difference in decision quality is always larger than the difference in costs, which are by assumption sufficiently small; formally: $\Pi(\sigma^*) - \Pi(\sigma') > c + f \implies u_i(\sigma^*) > u_i(\sigma')$ for all $i$. If $\sigma'$ is not minimal, there is a player who can save costs without affecting decision quality and utility of other shareholders.

By Claim 1 and 2 together, each Pareto-efficient strategy profile is an equilibrium with maximal decision quality. Clearly, there exists an equilibrium, say $\sigma^*$, with maximal decision quality. Now consider any Pareto-efficient equilibrium $\sigma$, i.e., an equilibrium that is not Pareto-dominated by any other equilibrium. This equilibrium must also maximize decision quality, i.e., $\sigma \in S^{\text{max}}$. Otherwise, it would be dominated by $\sigma^*$, as higher decision quality means strictly higher utility for every shareholder (again due to the small cost assumption). Therefore, every Pareto-efficient equilibrium must maximize decision quality.

Let us now turn to the model without a PA. Let $T$ be the set of all pure strategy profiles (without a PA). In full analogy to above, we define $T^*$ as the set of strategy profiles that are maximizing decision quality and that are minimal. Now observe that any strategy profile in $T$ (without a PA) corresponds to a strategy profile in $S$ (with a PA) where simply no player subscribes to the PA’s vote recommendation. Consequently, any decision quality obtained with a strategy profile in $T$ can also be obtained with a strategy profile in $S$. Let $\tilde{\sigma} \in S$ be a strategy profile that mimicks the maximal decision quality obtainable without a PA. Let $\Pi^{\text{with-PA}}$, respectively $\Pi^{\text{no-PA}}$, denote the maximal decision quality in the framework with a PA, respectively without a PA (for any strategy profile in the corresponding games). Since in the Pareto-efficient Nash equilibria with a PA, decision quality is maximal, we have $\Pi(\sigma^*) = \Pi^{\text{with-PA}} \geq \Pi(\tilde{\sigma}) = \Pi^{\text{no-PA}}$ for any Pareto-efficient equilibrium $\sigma^*$ in the game with a PA.

---

50Indeed, there are strategy profiles where all strategies are minimal, but still some players can reduce their information-acquisition without affecting the decision. For instance, when $N - 1$ players play NotSubscribe-NotInvest and vote yes as if in Rubber-stamping and one player does not subscribe to the PA but unconditionally invests into an own signal, as in UNIS. If the latter player stops acquiring an own signal, decisions are unaffected because she is never pivotal. By our definition, such strategy profiles are not minimal.

51There are only four strategies for each player: as in Rubber-stamping, as in Protest, as in UNIS, and voting contrary to the own signal.
References


Supplementary Online Material (SOM) for “When do proxy advisors improve corporate decisions?”

1 All Symmetric Equilibria
   1.1 Remaining Symmetric Equilibria in Benchmark Setting without a Proxy Advisor
   1.2 Remaining Symmetric Equilibria with an Early Proxy Advisor
   1.3 All Symmetric Equilibria with a Late Proxy Advisor

2 Asymmetric Equilibria
   2.1 Asymmetric Equilibria in Benchmark Setting without a Proxy Advisor
   2.2 Lemmata for Asymmetric Equilibria with a Proxy Advisor
   2.3 Research Incentives Increase with a Proxy Advisor
   2.4 Decision Quality Improves with a Proxy Advisor

These Supplementary Online Materials are available at http://bit.ly/proxy-SOM.
1 All Symmetric Equilibria

In the main text, the equilibrium analysis was restricted to the Assumptions BIB and PAF, i.e., that the board is better informed than a single shareholder and that proxy advice arrives before shareholders decide on own research. In this section we characterize all symmetric equilibria with and without these two assumptions. An overview of all symmetric equilibria is provided in SOM Table 1.1.

<table>
<thead>
<tr>
<th>Information-acquisition strategy</th>
<th>No PA Benchmark</th>
<th>With early PA Under PAF</th>
<th>With late PA Violating PAF</th>
</tr>
</thead>
<tbody>
<tr>
<td>NotSubscribe-NotInvest</td>
<td>Rubber*, Protest (UNIS*)</td>
<td>Rubber*, Protest (UNIS*)</td>
<td>Rubber*, Protest (UNIS*)</td>
</tr>
<tr>
<td>NotSubscribe-Invest</td>
<td>--</td>
<td>--</td>
<td>--</td>
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<tr>
<td>Subscribe-NotInvest</td>
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<tr>
<td>Subscribe-Invest</td>
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<tr>
<td>Subscribe-InvestIFF for</td>
<td>(Cand. 5a), (Cand. 5b)</td>
<td>CAIS*, CAIS-2</td>
<td></td>
</tr>
<tr>
<td>Subscribe-InvestIFF against</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 1.1: All symmetric equilibria arranged by information-acquisition strategy. Equilibria in brackets are precluded by Assumption BIB. Equilibria marked by "∗" are Pareto-efficient in some area of the parameter space. The "--" indicates that there are symmetric strategy profiles with this information-acquisition strategy, but none of them is an equilibrium; in contrast to the empty cells which indicate that these information-acquisition strategies cannot be played due to the setting.

When there is no PA there are three symmetric equilibria. UNIS, in which all shareholders invest in research, is restricted to an area of the parameter space where Assumption BIB is violated. Shareholders who do not invest in research can play Rubber-stamping or do the opposite: vote no unconditionally, which we call Protest. Both these symmetric strategy profiles are trivial equilibria, as no shareholder is ever pivotal and they incur no costs. Protest induces a decision quality $\Pi(\sigma) = 1 - q_B$ because it leads to the correct decision whenever the board’s proposal is wrong. Clearly, it is Pareto-dominated by Rubber-stamping, as $q_B > 0.5 > 1 - q_B$ and both induce the same costs (none).

These three equilibria also exist when there is a PA (last two columns of SOM Table 1.1) and their discussion is analogous. With information-acquisition strategy Subscribe-InvestIFF for, there are two additional symmetric equilibria, labelled Cand. 5a and Cand. 5b. However, they are both precluded by Assumption BIB and moreover Pareto-dominated (in fact by UNIS). Moreover, there are two additional equilibria, based on information-acquisition strategy Subscribe-InvestIFF against. One of them is CAIS. The other equilibrium, CAIS-2, only differs from CAIS in the voting behavior when the vote recommendation is for. In CAIS shareholders vote yes, while shareholders in CAIS-2 vote no, i.e., they do not approve the board’s proposal when the PA recommends to. CAIS-2 is Pareto-dominated by CAIS since it induces the same costs, but a lower decision quality than CAIS.

More striking than the additional equilibria which are Pareto-dominated is the observation that there are no equilibria with information-acquisition strategies Subscribe-NotInvest and Subscribe-Invest (independent of Assumptions BIB and PAF). To see why not, note
that buying only the vote recommendation, i.e., information-acquisition strategy Subscribe-NotInvest, is only worthwhile when using this information in an instance of pivotality. However, if all shareholders symmetrically use the vote recommendation, then no shareholder is ever pivotal. Similarly, acquiring both signals, i.e., information-acquisition strategy Subscribe-Invest, is worthwhile only if shareholders condition their vote on both PA advice and own signal such that none is superfluous, e.g., by voting yes if and only if one of the latter is in favor of the proposal. When all shareholders adopt this strategy, pivotality already implies that the recommendation was against. Hence, saving the subscription fee by not subscribing to the PA is a profitable deviation.

Remember that Lemmas A.1 and A.2 in the Appendix of the main text provide all symmetric strategy profiles that can be equilibria, together with their parameter conditions and decision quality under the Assumptions BIB and PAF. We now provide the corresponding results for all remaining equilibria that were excluded by Assumptions BIB and PAF -- first, for the benchmark case that there is no PA, then for a PA whose advice arrives early, i.e., under Assumption PAF, finally for the case that there is a PA whose advice arrives later, i.e., violating Assumption PAF. In SOM Table 1.1, this means that the main text has already addressed the first two columns with the equilibria that are not in brackets and we now complete the analysis by addressing the equilibria in brackets of the first column in SOM Section 1.1, then the equilibria in brackets of the second column in SOM Section 1.2; and finally all equilibria of the last column in SOM Section 1.3.

1.1 Remaining Symmetric Equilibria in Benchmark Setting without a Proxy Advisor

**Lemma 1.1** (SYM without PA: All Remaining Equilibria). Suppose no PA is admitted. In addition to the equilibria provided in Lemma A.1, there is the following symmetric equilibrium when Assumption BIB is relaxed:

i. **UNIS**, i.e., all shareholders invest in own research and vote according to their signal, is a symmetric equilibrium if and only if $q_S > q_B$. Its decision quality is $\Pi(\sigma^{UNIS}) = \pi(N)$.

**Proof.** Suppose first that $q_S \leq q_B$ (i.e., Assumption BIB holds). Then UNIS is not an equilibrium as shown by Lemma A.1 (in the Appendix A of the main text).

Now, suppose $q_S > q_B$. In order to show that UNIS is an equilibrium, we show that there is no individual deviation that improves utility. We use the following principle: if a deviation is more attractive than an other deviation in terms of utility, then excluding the former is sufficient to exclude the latter. We organize the potential deviations by information-acquisition strategy. As we consider the setting without a PA, there are only two information-acquisition strategies: acquiring an own signal or not. Pivotality always implies that among the $N - 1$ other shareholders the signals are split in $(N-1)/2$ a-signals and $(N-1)/2$ b-signals, which occurs with positive probability.

1. When no own signal is acquired, strategies are Rubber-stamping and Protest.

Consider first the deviation from UNIS to Rubber-stamping. When pivotal, voting yes would weakly increase decision quality if $q_B \geq q_S$. Rubber-stamping decreases decision
quality as \( q_S > q_B \) holds by assumption. Rubber-stamping saves costs \( c \). For small enough \( c \) (as it is assumed in the lemma), this deviation does not increase utility.

The deviation to always vote no without information acquisition (Protest) changes the outcome to no in case of pivotality. It would then induce A despite the fact that, conditional on pivotality, all other shareholder’s signals balance each other and that the board’s signal is \( b \). Hence, decision quality is affected in a worse way than with Rubber-stamping, while cost savings are equal. Therefore this latter deviation is not as attractive as the deviation to Rubber-stamping.

2. When a signal is acquired, the information-acquisition strategy is unchanged. Deviation to a different voting strategy is not an improvement. First, not conditioning on the acquired signal is less attractive than the deviation to Rubber-stamping or to unconditionally voting no, as it involves higher costs. Conditioning on the own signal leaves one deviation on the voting stage: vote yes if \( a \) and no if \( b \), which is the opposite of UNIS. However, if voting yes after \( a \) (against) was optimal in case of pivotality, then voting no after \( a \) would also be. Hence, shareholders could improve by unconditionally voting no.

We have established that Rubber-stamping is the most attractive deviation. For \( q_B > q_S \), the deviation to Rubber-stamping strictly decreases decision quality, and hence does not increase utility for low enough costs \( c \).

Finally, the decision quality of UNIS equals \( \pi(N) \) because the signal that has been received by a majority of the \( N \) (odd) voters determines the decision. Hence, the ex ante probability that the decision matches the true state equals the probability that among \( N \) independent signals of quality \( q_S \) the majority is correct, which is the definition of \( \pi(N) \).

Let us now show that there are no further symmetric equilibria. There are only two information-acquisition strategies. For not investing in an own signal both strategy profiles are symmetric equilibria, as already addressed in parts i. and ii. of Lemma A.1). Consider now investment in an own signal. Since shareholders pay \( c \) they must condition their vote on their own signal. Otherwise, they could improve by voting in the same way and not investing \( c \). Conditioning on their signal leaves two pure strategies: vote yes if \( b \) and no if \( a \) (i.e., UNIS) or the opposite (vote yes if \( a \) and no if \( b \)). If voting yes after \( a \) (against) was optimal, then voting no after \( a \) would also be. Hence, shareholders could improve by unconditionally voting A. Only UNIS remains when shareholders acquire an own signal.

\[ \square \]

1.2 Remaining Symmetric Equilibria with an Early Proxy Advisor

Lemma 1.2 (SYM with PA under Assumption PAF: All Remaining Equilibria). Let Assumption PAF hold. Let costs \( c \) be arbitrarily small and let fee \( f \) be sufficiently smaller. In addition to the equilibria provided in Lemma A.2, there are the following symmetric equilibria when Assumption BIB is relaxed:

i. UNIS is a symmetric equilibrium if and only if \( \ell_S > \ell_B + \ell_P \). Its decision quality is \( \Pi(\sigma^{UNIS}) = \pi(N) \).
ii. Cand. 5a (i.e., shareholders subscribe to PA and invest in own research iff the vote recommendation is for, i.e., Subscribe-InvestIFF for; when the recommendation is for, they vote yes iff their own signal is b; when the recommendation is against, they vote yes), as illustrated in Table A.4, is a symmetric equilibrium if and only if \( \ell_S > \ell_B + \ell_P \). Its decision quality: 
\[
\Pi(\sigma^{\text{Cand. 5a}}) = q_B(1 - q_P) + [q_B q_P + (1 - q_B)(1 - q_P)](\pi(N)).
\]

iii. Cand. 5b (i.e., shareholders subscribe to PA and invest in own research iff the vote recommendation is for, i.e., Subscribe-InvestIFF for; when the recommendation is for, they vote yes iff their own signal is b; when the recommendation is against, they vote no), as illustrated in SOM Table 1.2, is a symmetric equilibrium if and only if \( \ell_S > \ell_B + \ell_P \). Its decision quality is 
\[
\Pi(\sigma^{\text{Cand. 5b}}) = (1 - q_B)q_P + [q_B q_P + (1 - q_B)(1 - q_P)](\pi(N)).
\]

Proof. We address each part of SOM Lemma 1.2 separately.

i. Suppose first that \( \ell_S \leq \ell_B + \ell_P \). We show that there is a utility improving deviation.

Consider shareholder \( i \) who deviates to strategy CAIS. (CAIS is illustrated in Table 1.) A deviation to CAIS differs from UNIS only when the vote recommendation is for and the own signal is \( a \): with UNIS she would vote no, with CAIS she votes yes. It weakly improves decision quality iff \( \ell_B + \ell_P \geq \ell_S \), which we assumed at the beginning of this argument. It saves costs if \( f < c[q_B q_P + (1 - q_B)(1 - q_P)] \), which is satisfied by assumption that \( f \) is sufficiently lower than \( c \). Hence, \( i \) improves utility by deviating to CAIS.

Suppose now that \( \ell_S > \ell_B + \ell_P \). In order to show that UNIS is an equilibrium, we show that there is no individual deviation that improves utility. We use the following principle: if a deviation is more attractive than an other deviation in terms of utility, then excluding the former is sufficient to exclude the latter. We organize the potential deviations by information-acquisition strategy. Pivotality always implies that among the \( N - 1 \) other shareholders the signals are split in \( \frac{N - 1}{2} \) \( a \)-signals and \( \frac{N - 1}{2} \) \( b \)-signals, which occurs with positive probability.

(1) NotSubscribe-NotInvest. Deviating to Rubber-stamping decreases decision quality, as \( \ell_S > \ell_B + \ell_P \) implies that \( q_S > q_B \). It saves costs \( c \). Hence, the deviation to Rubber-stamping is not beneficial for low enough \( c \). Deviation to unconditionally voting no is even less attractive than deviating to Rubber-stamping (see also Proof of SOM Lemma 1.1).

(2) NotSubscribe-Invest. A deviation to using the same information-acquisition strategy as in UNIS (NotSubscribe-Invest) but a different voting strategy is not an improvement (see Proof SOM Lemma 1.1).

(3) Subscribe-NotInvest. Deviating to subscribe to the PA without investing into an own signal (Subscribe-NotInvest) is most attractive when voting according to the PA’s recommendation. (When not conditioning on the vote recommendation, the shareholder could better deviate to no information acquisition, i.e., NotSubscribe-NotInvest.) Voting according to the PA’s recommendation weakly improves decision

\[52\] We use this assumption here only for the special case \( \ell_B + \ell_P = \ell_S \).
quality iff \( q_P \geq q_S \), which is precluded by \( \ell_S > \ell_B + \ell_P \). Hence, this deviation strictly decreases decision quality. Consider now the costs. The deviation costs an additional \( f \) but saves \( c \). If costs \( c \) are sufficiently low (as it is assumed in SOM Lemma 1.2), the decision quality difference dominates any cost difference, and hence, this deviation does not increase \( i \)'s utility.

(4) Subscribe-Invest. Deviating to subscribe to the PA and invest into an own signal requires conditioning on both the vote recommendation and the own signal. Otherwise, there are other deviations which are more attractive because of the costs. Two cases are possible.

Case 1 is the deviation illustrated in Table A.2, i.e., voting \( \text{yes} \) except when both the own signal and the PA’s recommendation contradict the board’s proposal, then the deviating shareholder votes \( \text{no} \). This deviation cannot alter the outcome after recommendation \( \text{against} \) because voting according to one’s signal is like in UNIS. The difference to UNIS occurs if the vote recommendation is \( \text{for} \) and \( i \)'s signal is \( a \) (in UNIS \( i \) would vote \( \text{no} \), in this deviation she would vote \( \text{yes} \)). Conditional on that case and on pivotality, the deviation weakly improves decision quality iff \( \ell_B + \ell_P \geq \ell_S \), which is precluded by assumption \( \ell_S > \ell_B + \ell_P \). Considering that the deviation is more costly than UNIS, it is not beneficial.

The second case is illustrated in Table A.3, i.e., voting \( \text{no} \) except when both the own signal and the PA’s recommendation are aligned with the board’s proposal. This deviation cannot alter the outcome after recommendation \( \text{for} \) because voting according to one’s signal is like in UNIS. The difference to UNIS hence only occurs if the vote recommendation is \( \text{against} \) and the signal is \( b \): in UNIS \( i \) would vote \( \text{yes} \); in this deviation she would vote \( \text{no} \). Conditional on that case and on pivotality, the deviation weakly improves decision quality iff \( \ell_P \geq \ell_S + \ell_B \), which is precluded by assumption \( \ell_S > \ell_B + \ell_P \). Hence, this deviation is not beneficial, considering that it is also more costly than UNIS.

(5) Subscribe-Invest iff \( \text{for} \). Consider the deviation to subscribing to the PA and investing into an own signal iff the PA’s recommendation is \( \text{for} \).

Since shareholders pay \( f \) and sometimes \( c \) they must condition their voting strategy on the information they have acquired. (Otherwise, there would be a more attractive deviation without these costs.) In particular, they must vote according to their signal after the \( \text{for} \) recommendation. There are two cases. They correspond to the strategies Cand. 5a and Cand. 5b.

Cand. 5a: shareholders vote \( \text{yes} \) except if the vote recommendation is \( \text{for} \) and the own signal is \( a \) (against) as in Table A.4. The deviation to Cand. 5a cannot alter the outcome after recommendation \( \text{for} \) because voting according to one’s signal is like in UNIS. The difference to UNIS hence only occurs if the vote recommendation is \( \text{against} \) and the signal is \( a \) (in UNIS \( i \) would vote \( \text{no} \), in this deviation she would vote \( \text{yes} \)). Conditional on that case and on pivotality, the deviation would weakly improve decision quality iff \( \ell_B \geq \ell_S + \ell_P \). Since \( \ell_S > \ell_B + \ell_P \) by assumption, we have \( \ell_B < \ell_S + \ell_P \). Hence, this deviation decreases decision quality. The deviation costs \( f \) with certainty and \( c \) with probability \( q_B q_P + (1-q_B) (1-q_P) \). UNIS costs \( c \) with certainty. The cost difference (costs of UNIS minus costs of Cand. 5b) is
hence \(c[q_B(1-q_P) + (1-q_B)q_P] - f\) which is small for \(c\) arbitrarily small (as it is assumed in the lemma). Hence, the deviation does not increase utility of a deviating shareholder.

Cand. 5b: shareholders vote \textit{no} except if the vote recommendation is \textit{for} and the own signal is \(b\) (for board) as in SOM Table 1.2.

<table>
<thead>
<tr>
<th>Own Signal</th>
<th>(b) (for board)</th>
<th>(a) (against)</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textit{for}</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>\textit{against}</td>
<td>no</td>
<td></td>
</tr>
</tbody>
</table>

\textbf{Table 1.2:} Cand. 5b: A strategy based on acquiring an own signal iff the PA’s recommendation is \textit{for}: Subscribe-InvestIFF \textit{for} and vote \textit{no}, except if the PA’s recommendation is \textit{for} and the own signal is \(b\).

The deviation to Cand. 5b cannot alter the outcome after recommendation \textit{for} because voting according to one’s signal is like in UNIS. The difference to UNIS hence only occurs if the vote recommendation is \textit{against} and the signal is \(b\) (in UNIS \(i\) would vote \textit{yes}, in this deviation she would vote \textit{no}). Conditional on that case and on pivotality, the deviation would weakly improve decision quality iff \(\ell_P \geq \ell_S + \ell_B\). By assumption \(\ell_S > \ell_B + \ell_P\), we have \(\ell_P < \ell_S + \ell_B\). Hence, this deviation reduces decision quality. The cost difference (costs of UNIS minus costs of Cand. 5b) is again \(c[ q_B (1-q_P) + (1-q_B)q_P] - f\), which is small for \(c\) arbitrarily small (as it is assumed in the lemma). Hence, the deviation does not increase utility of a deviating shareholder.

(6) Subscribe-InvestIFF \textit{against}. Consider a deviation to subscribing to the PA and investing into an own signal iff the vote recommendation is \textit{against}.

Since shareholders pay \(f\) and sometimes \(c\) they must condition their voting strategy on the information they have acquired. (Otherwise, there would be a more attractive deviation, saving the costs.) In particular, they must vote according to their signal after the \textit{against} recommendation. Two cases remain: CAIS as illustrated in Table 1 and CAIS-2 as illustrated in Table A.1. A deviation to CAIS differs from UNIS only when the vote recommendation is \textit{for} and the own signal is \(a\) (with UNIS she would vote \textit{no}, with CAIS she votes \textit{yes}). It would weakly improve decision quality iff \(\ell_B + \ell_P \geq \ell_S\). Since \(\ell_S > \ell_B + \ell_P\), it strictly decreases decision quality. The cost difference (costs of UNIS minus costs of CAIS) is \(c[q_B q_P + (1-q_B)(1-q_P)] - f\) and decreases in \(c\). Hence, the difference in decision quality dominates, and the deviation to CAIS lowers the deviating shareholder’s utility lower.

Consider now a deviation to strategy CAIS-2 (which only differs from CAIS by voting \textit{no} when the vote recommendation is \textit{for}). It differs from UNIS only when the vote recommendation is \textit{for} and the own signal is \(b\) (indeed, with UNIS she votes \textit{yes}, with CAIS-2 she would vote \textit{no}). The deviation to CAIS-2 would weakly improve decision quality iff \(\ell_B + \ell_S + \ell_P \leq 0\), which is never satisfied. It decreases decision quality. Since we assume sufficiently low costs, this deviation is not profitable.
Now, we have covered all deviations that can occur. If $\ell_S > \ell_B + \ell_P$, UNIS is an equilibrium for low enough costs $c$.

ii. We have to show that Cand. 5a is a symmetric equilibrium if and only if $\ell_S > \ell_B + \ell_P$. The voting strategy is illustrated in Table A.4. A shareholder is not pivotal after the against recommendation, but after the for recommendation when all the other $N-1$ shareholder’s signals are exactly split.

Suppose first that $\ell_S \leq \ell_B + \ell_P$. We show that there is a beneficial deviation. Consider shareholder $i$ who deviates to Rubber-stamping. This deviation only alters the outcome when the vote recommendation is for, all other shareholders’ signals are split, and $i$’s signal is $a$ (against): Under Cand. 5a, $i$ would vote no, but under Rubber-stamping she votes yes. Decision quality weakly improves by this deviation given $\ell_B + \ell_P \geq \ell_S$. Moreover, costs are lower. Hence, Cand. 5a cannot be an equilibrium.

Suppose now that $\ell_S > \ell_B + \ell_P$. In order to show that Cand. 5a is an equilibrium, we show that there is no individual deviation that improves utility. We use the following principle: if a deviation is more attractive than an other deviation in terms of utility, then excluding the former is sufficient to exclude the latter. We organize the potential deviations by information-acquisition strategy. Pivotality always implies that the vote recommendation is for and among the $N-1$ other shareholders the signals are split in $\frac{N-1}{2}$ $a$-signals and $\frac{N-1}{2}$ $b$-signals, which occurs with positive probability.

1. NotSubscribe-NotInvest. Deviating to Rubber-stamping, while saving costs, decreases decision quality as $\ell_B + \ell_P < \ell_S$. Hence, the deviation to Rubber-stamping is not beneficial for low enough $c$ and $f$.

   Deviating to unconditionally voting no (Protest) lowers decision quality in any case and is therefore less attractive than deviating to Rubber-stamping.

2. NotSubscribe-Invest. Consider deviating to information-acquisition strategy NotSubscribe-Invest. The most attractive combination of this strategy with a voting strategy is UNIS: UNIS conditions the vote on the acquired signal in a way that maximizes decision quality. However, a deviation to UNIS does not change the outcome, since in case of pivotality (when the vote recommendation is for) the voting strategies are identical. Cand. 5a is less expensive than UNIS if $f + [q_Bq_P + (1- q_B)(1- q_P)]c \leq c$, which is $f \leq c(1-[q_Bq_P + (1- q_B)(1- q_P)])$. This is satisfied for $f$ sufficiently lower than $c$. Hence, UNIS is not a profitable deviation.

3. Subscribe-NotInvest. A deviation to subscribing to the PA is most attractive when voting according to the recommendation. (Voting the opposite of the recommendation reduces decision quality. When not conditioning on the vote recommendation, the shareholder could save costs and deviate to no information acquisition, NotSubscribe-NotInvest.) Voting according to the recommendation alters the outcome if vote recommendation is for, other shareholder’s signals are split, and $i$’s signal is $a$ (against): With Cand. 5a, $i$ would vote no, with this deviation $i$ would vote yes. The deviation weakly improves decision quality iff $\ell_B + \ell_P \geq \ell_S$, which is precluded by $\ell_S > \ell_B + \ell_P$. It decreases decision quality. The deviation saves costs $c$, but for low enough $c$ it is not beneficial to deviate.
(4) Subscribe-Invest. A deviation to subscribing to the PA and investing into an
own signal requires to conditioning on both vote recommendation and own signal.
Otherwise, there are other deviations which are more attractive, producing the same
decision quality and saving the costs. Two cases are possible.
Case 1 is the deviation illustrated in Table A.2. The difference to UNIS only occurs
if the vote recommendation is for and the own signal is a (in UNIS i would vote no,
in this deviation she would vote yes). Conditional on that case and conditional on
pivotality, the deviation weakly improves decision quality iff \( \ell_B + \ell_P \geq \ell_S \), which
is precluded by assumption. Considering that the deviation is more costly than
Cand. 5a, this deviation is never beneficial.
The second case is illustrated in Table A.3. This deviation does not alter the outcome
(after vote recommendation for, voting is according to signal like in Cand. 5a, and
after recommendation against, the deviating shareholder is not pivotal). Considering
that the deviation is also more costly than Cand. 5a, this deviation is never beneficial.

(5) Subscribe-InvestIFF for. Consider a deviation that keeps information-acquisition
strategy Subscribe-InvestIFF for but changes the voting strategy. Changes after the
recommendation against are ineffective, as no shareholder is pivotal. Since share-
holders pay \( c \) after recommendation for they must condition their voting strategy
on the information they have acquired. (Otherwise, there would be a more attractive
deviation that produces the same decision quality but saves \( c \).) In particular, they
must condition their vote on their signal after the for recommendation. This means
that the deviation is voting yes when the own signal is a (against) and voting no
when the own signal is b (for board). This deviation leads to the same costs as
Cand. 5a but clearly reduces decision quality.

(6) Subscribe-InvestIFF against. Consider a deviating shareholder who subscribes to the
PA and invests into an own signal iff the PA’s recommendation is against. Since
the shareholder pays \( f \) and sometimes \( c \), she must condition her voting strategy
on the information she has acquired. (Otherwise, there would be a more attractive
deviation that produces the same decision quality but saves costs.) In particular,
she must vote according to her signal after the against recommendation. Two cases
remain.
Case 1 is CAIS as illustrated in Table 1. It alters the outcome of Cand. 5a only when
the vote recommendation is for and the own signal is a: Indeed, with Cand. 5a she
would vote no, with CAIS she votes yes. The deviation weakly improves decision
quality iff \( \ell_B + \ell_P \geq \ell_S \), which is precluded by assumption. It reduces decision
quality. It does save some cost: Cand. 5a costs \( f + [q_B q_P + (1 - q_B)(1 - q_P)]c \),
CAIS costs \( f + [q_B(1 - q_P) + (1 - q_B)q_P]c \). For low enough costs \( c \), CAIS is not an
improvement.
Case 2 is called CAIS-2 and illustrated in Table A.1 (it only differs from CAIS by
prescribing to vote no when the vote recommendation is for). CAIS-2 differs from
Cand. 5a only when the vote recommendation is for and the own signal is b (with
Cand. 5a the shareholder would vote yes, with CAIS-2 she votes no). The deviation
would weakly improve decision quality iff \( \ell_B + \ell_S + \ell_P \leq 0 \), which is precluded by
assumption. It reduces decision quality. Hence, even for small enough costs c, this is not a beneficial deviation.

Now, we have covered all deviations that can occur. If $\ell_S > \ell_B + \ell_P$, Cand. 5a is an equilibrium for low enough costs c and sufficiently lower f.

The decision quality of the equilibrium Cand. 5a amounts to $q_B q_P \pi(N) + q_B (1 - q_P) * 1 + (1 - q_B) q_P * 0 + (1 - q_B) (1 - q_P) * \pi(N)$.

iii. The proof that Cand. 5b is an equilibrium under the same conditions as Cand. 5a is identical to the proof for Cand. 5a (immediately above).

The decision quality of the equilibrium Cand. 5b amounts to $q_B q_P \pi(N) + q_B (1 - q_P) * 0 + (1 - q_B) q_P * 1 + (1 - q_B) (1 - q_P) * \pi(N)$.

To show that there are no additional equilibria, we exhaustively discuss all pure strategies. Again, we organize the discussion by information-acquisition strategy.

(1) NotSubscribe-NotInvest. There are only voting strategies yes or no. Both lead to equilibria as shown in parts i. and ii. of Lemma A.1. (Assumption BIB does not matter for these results.)

(2) NotSubscribe-Invest. Since shareholders pay c they must condition on their own signal. Otherwise, they could improve by voting in the same way and not investing c. Conditioning on the own signal leaves two pure strategies: voting yes if the signal is b and voting no if the signal is a (i.e., UNIS) or the opposite (voting yes if the signal is a and voting no if the signal is b). If voting yes after the own signal is a (against) was optimal, then voting no after signal a would also be so. Hence, shareholders could improve by unconditionally voting no. Only UNIS remains. When UNIS is an equilibrium has been already addressed in this lemma, SOM Lemma 1.2, part i.

(3) Subscribe-NotInvest. Since shareholders pay f, they must condition on the PA’s recommendation. Hence, they either vote yes after for and no after against, or they do the opposite. In either case, no shareholder is pivotal since all votes are the same given one particular vote recommendation. A shareholder can improve by not paying f and voting unconditionally, e.g., yes. Hence, there is no symmetric equilibrium with this information-acquisition strategy.

(4) Subscribe-Invest. Since shareholders pay both f and c, they must condition their voting strategy on both the vote recommendation and the own signal. Otherwise, they could improve their utility by exhibiting the same voting behavior, but saving costs. This means that only two voting strategies remain.

Case 1: Consider the strategy to vote yes except if the vote recommendation is against and the signal is a, as in Table A.2. In this case no shareholder is pivotal if the PA recommends for (as the recommendation is common for all shareholders). Hence, shareholder i can only be pivotal if the vote recommendation is against. If so, i votes according to her own signal. Hence, deviating to UNIS would not change the outcome because either i is not pivotal or i would also vote according to the own signal. However,
UNIS saves fee $f$ and is hence a profitable deviation. Thus, the strategy profile of case 1, illustrated in Table A.2, cannot be a symmetric equilibrium.

Case 2: Consider the strategy to vote no except if the vote recommendation is for and the own signal is $b$ (for board), as in Table A.3. The analogous argument as above for case 1 applies, as follows: In this case no shareholder is pivotal if the PA recommends against (as the recommendation is common for all shareholders). Hence, shareholder $i$ can only be pivotal if the recommendation is for. If so, $i$ votes according to the own signal. Hence, deviating to UNIS would not change the outcome because either $i$ is not pivotal or $i$ would also vote according to the own signal. However, UNIS saves fee $f$ and is hence a profitable deviation. Thus, the strategy profile of case 2 cannot be a symmetric equilibrium.

Therefore, there cannot be a symmetric equilibrium with this information-acquisition strategy (Subscribe-Invest), in which shareholders unconditionally buy both PA’s recommendation and own signal.

(5) Subscribe-Investiff for. Since shareholders pay $f$ and sometimes $c$, they must condition their voting strategy on the vote recommendation and the own signal when they acquire them. In particular, after having bought the own signal on top of the recommendation for, shareholders must vote according to their signal in equilibrium. Voting the opposite is dominated and not conditioning as well. This leaves two cases, which we have already addressed as Cand. 5a and Cand. 5b in this lemma (SOM Lemma 1.2) in parts ii. and iii.

(6) Subscribe-Investiff against. Since shareholders pay $f$ and sometimes $c$, they must condition their voting strategy on the vote recommendation and the own signal when they acquire them. In particular, after having bought the own signal on top of the recommendation against, shareholders must vote according to their signal in equilibrium. Voting the opposite is dominated and not conditioning as well. This leaves two cases: CAIS and CAIS-2, which we have already addressed in Lemma A.2. (Assumption 1 does not affect these equilibria.)

Hence, there are no further symmetric equilibria.

\[\square\]

1.3 All Symmetric Equilibria with a Late Proxy Advisor

Consider now the situation when a PA is admitted and proxy advice arrives after the shareholders’ decision to invest in own research. That is, all actions occur as illustrated in the timeline (Figure 1), but proxy advice arrives at the end of period $t = 2$. This timeline admits information-acquisition strategies NotSubscribe-NotInvest, NotSubscribe-Invest, Subscribe-NotInvest, and Subscribe-Invest, which are all also playable when Assumption PAF is satisfied. However, the violation of Assumption PAF precludes the two information-acquisition strategies Subscribe-Investiff for and Subscribe-Investiff against, the latter of which was crucial for our main results. SOM Table 1.1 illustrates this and already indicates that in this setting, the equilibria are very similar to the setting where there is no PA.
Comparing these two settings (the benchmark setting of no PA with a PA whose recommendation arrives late) we note that the presence of a PA who violates Assumption PAF admits two additional strategies: Subscribe-NotInvest, and Subscribe-Invest. It turns out that none of these two additional information-acquisition strategies is part of an equilibrium. Intuitively, buying only the vote recommendation, i.e., information-acquisition strategy Subscribe-NotInvest, is only worthwhile when using this information in an instance of pivotality. However, if all shareholders symmetrically use the vote recommendation, then no shareholder is ever pivotal. If they do not use it, they could save costs by not paying the fee \( f \). Similarly, and more importantly, acquiring both signals, i.e., information-acquisition strategy Subscribe-Invest, is never part of an equilibrium. The reason is that shareholders who acquire both the PA’s signal and an own signal must condition their vote on both such that none is superfluous, e.g., by voting yes if and only if one of them is in favor of the proposal. When all shareholders adopt such a strategy, pivotality already implies the content of the vote recommendation. For instance, if shareholders vote yes if and only if either the vote recommendation or the own signal is in favor of the proposal, pivotality implies that the recommendation was against. Hence, there is a deviation to not paying the subscription fee. Analogous arguments exist for every other possible voting strategy that conditions on combinations of PA advice and the own signal. Therefore, admitting a late PA does not lead to additional equilibria. In contrast, it may destroy equilibria that existed without a PA because it offers additional deviation possibilities. This in fact occurs for some parameter range, as illustrated in SOM Figure 1.1 in the left area above the triangle, where UNIS ceases to be an equilibrium, while Rubber-stamping becomes the Pareto-efficient equilibrium. SOM Proposition 1.1 provides the general result.

![Diagram](image)

**Figure 1.1:** Pareto-efficient symmetric equilibria with late PA, i.e., when Assumption PAF is violated.

**Proposition 1.1** (SYM with PA violating Assumption PAF). Let costs \( c \) be arbitrarily small. Suppose there is a PA whose vote recommendation arrives after shareholders decided upon own research, i.e., Assumption PAF is violated.
i. If $\ell_S \geq \ell_B + \ell_P$, then there exists a symmetric equilibrium in which shareholders invest in own research. The Pareto-efficient equilibrium is UNIS and leads to decision quality $\Pi(\sigma^{UNIS}) = \pi(N)$.

ii. Otherwise, there does not exist a symmetric equilibrium in which shareholders invest in own research. The Pareto-efficient equilibrium is Rubber-stamping and leads to decision quality $\Pi(\sigma^{rubber}) = q_B$.

Proof. To show part i. of SOM Proposition 1.1, we use SOM Lemma 1.3 below, which shows that UNIS is an equilibrium (part iii.) and that there are two further equilibria: Rubber-stamping and Protest. It remains to show that UNIS Pareto-dominates rubber-stamping and the other trivial equilibrium (Protest) for $\ell_S \geq \ell_B + \ell_P$. We have

$$\Pi(\sigma^{UNIS}) = \pi(N) > q_S > q_B = \Pi(\sigma^{Rubber}) > 1 - q_B = \Pi(\sigma^{Protest}),$$

where $\sigma^{Protest}$ stands for the equilibrium in which shareholders acquire no information (NotSubscribe-NotInvest) and vote no unconditionally. Now, for costs $c$ low enough (as it is assumed in the proposition), UNIS Pareto-dominates because of its higher decision quality.

To show part ii. of SOM Proposition 1.1, we use again SOM Lemma 1.3. For $\ell_S < \ell_B + \ell_P$, UNIS is not an equilibrium (by SOM Lemma 1.3) and only two equilibria remain, Rubber-stamping and Protest. Rubber-stamping Pareto-dominates because it leads to higher decision quality $\Pi(\sigma^{Rubber}) = q_B > 0.5 > 1 - q_B = \Pi(\sigma^{Protest})$.

It is important to observe that Assumption BIB, $q_S \leq q_B$, implies $\ell_S < \ell_B + \ell_P$, which precludes UNIS here. Hence, we can summarize.

Remark. Under Assumption BIB, unconditional information acquisition neither occurs with late PA (SOM Proposition 1.1), nor with an early PA (SOM Proposition 1.2). Conditional information acquisition occurs with an early PA (Proposition 2), but not with a late PA (SOM Proposition 1.1).

Considering the benchmark setting without a PA, SOM Proposition 1.1 is very similar to Proposition 1, but differs in the condition for the two equilibria. In fact, the condition for an equilibrium with information acquisition becomes more demanding when a PA is admitted: $\ell_S \geq \ell_B + \ell_P$ means that a single shareholder has to be better informed, not only than the board, but than both the board and the PA together. The reason is that there is an additional deviation possibility, compared to the setting without a PA: A shareholder could invest in an own signal and buy the vote recommendation and then vote no only if both the vote recommendation and the own signal are against the board.\textsuperscript{53} This deviation only changes the voting outcome, compared to the UNIS strategy profile, if $i$ is pivotal and the PA’s recommendation is for and $i$’s signal is a: in UNIS $i$ votes no, in this deviation she

\textsuperscript{53}This strategy is illustrated in Table A.2.
would vote yes. Conditional on that case, the deviation improves decision quality iff

\[ q_B q_P (1 - q_S) \left( \frac{N - 1}{2} \right) q_S^{\frac{N - 1}{2}} (1 - q_S)^{\frac{N - 1}{2}} > (1 - q_B) (1 - q_P) q_S \left( \frac{N - 1}{2} \right) (1 - q_S)^{\frac{N - 1}{2}} q_S^{\frac{N - 1}{2}} \]

\[ q_B q_P (1 - q_S) > (1 - q_B) (1 - q_P) q_S \]

\[ \frac{q_B}{1 - q_B} \frac{q_P}{1 - q_P} > \frac{q_S}{1 - q_S} \]

\[ \ell_B + \ell_P > \ell_S. \]

When this condition is satisfied, then for a sufficiently small fee \( f \), the deviation is an improvement for \( i \) since it increases decision quality. Hence, \( \ell_S \geq \ell_B + \ell_P \) is a necessary condition for UNIS to be an equilibrium when costs \( c \) and fee \( f \) are arbitrarily small.

The foundation for SOM Proposition 1.1 is SOM Lemma 1.3 that we establish next.

**Lemma 1.3 (SYM with PA violating Assumption PAF: All Equilibria).** Let costs \( c \) be arbitrarily small and let fee \( f \) be sufficiently smaller. Suppose there is a PA whose vote recommendation arrives after shareholders decided upon own research, i.e., Assumption PAF is violated. Then in contrast to Lemma A.2 and SOM Lemma 1.2, we have:

i. Protest (i.e., no shareholder invests in own research and all shareholders vote no) is a symmetric equilibrium for any \( \ell_B, \ell_S \in (0, \infty) \). Its decision quality is \( 1 - q_B \).

ii. Rubber-stamping (i.e., no shareholder invests in research and all shareholders vote yes) is a symmetric equilibrium for any \( \ell_B, \ell_S \in (0, \infty) \). Its decision quality is \( q_B \).

iii. UNIS is a symmetric equilibrium if and only if \( \ell_S \geq \ell_B + \ell_P \). Its decision quality is \( \Pi(\sigma^{\text{UNIS}}) = \pi(N) \).

iv. There are no other symmetric equilibria. In particular, there is no equilibrium in which all shareholders subscribe to proxy advice and invest in own signal (Subscribe-Invest).

**Proof.** We address each part of SOM Lemma 1.3 separately.

i. The proof is identical to the proof of Lemma A.1, part i.

ii. The proof is identical to the proof of Lemma A.1, part ii.

iii. Suppose first that \( \ell_S < \ell_B + \ell_P \). We show that there is a beneficial deviation for \( f \) low enough.

Consider shareholder \( i \) deviates to the following strategy \( \sigma_i' \): subscribe and invest (Subscribe-Invest) and vote yes, except if the PA’s recommendation is against and the own signal is \( a \). This strategy is illustrated Table A.2.

Pivotality in UNIS implies that among the \( N - 1 \) other shareholders the signals are split in \( \frac{N - 1}{2} \) \( a \)-signals and \( \frac{N - 1}{2} \) \( b \)-signals. This deviation \( \sigma_i' \) does not alter the outcome after recommendation against because voting according to signal is like in UNIS. The difference to UNIS only occurs if the recommendation is for and \( i \)’s signal is \( a \): in UNIS \( i \) would vote no, in this deviation she would vote yes. Conditional on that case

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and conditional on pivotality, the deviation improves decision quality if and only if \( \ell_B + \ell_P > \ell_S \), which holds by the assumption made at the beginning of this argument. Note that the deviation has the same research costs \( c \) but additional costs \( f \). Hence, for \( f \) small enough (as it is assumed in this lemma) it strictly improves utility of \( i \).

Suppose now that \( \ell_S \geq \ell_B + \ell_P \). In order to show that UNIS is an equilibrium, we have to show that there is no individual deviation that improves utility. In the proof of SOM Lemma 1.2, part i., we consider all possible deviations from UNIS and show that there is no individual deviation that improves utility for \( \ell_S > \ell_B + \ell_P \). Here, there are less deviation possibilities to consider, as Subscribe-InvestIFF for and Subscribe-InvestIFF against cannot be played. For the other four information-acquisition strategies, all arguments from the proof of SOM Lemma 1.2 still apply under the assumption \( \ell_S \geq \ell_B + \ell_P \) when simply using that \( \ell_S \geq \ell_B + \ell_P \) implies \( q_S > q_B \) and \( q_S > q_P \).

iv. There are four information-acquisition strategies to consider.

1. For NotSubscribe-NotInvest, i.e., shareholders neither subscribe to the PA nor invest in an own signal, both strategy profiles are symmetric equilibria, as already addressed in parts i. and ii. of this Lemma, SOM Lemma 1.3.

2. For NotSubscribe-Invest, i.e., shareholders do not subscribe to the PA but invest in an own signal, UNIS is an equilibrium as established in SOM Lemma 1.3, part i. The other strategy with NotSubscribe-Invest, voting the opposite of the signal, is not an equilibrium. The proof is the same as in SOM Lemma 1.1.

3. Consider Subscribe-NotInvest, i.e., shareholders subscribe to the PA but do not invest in an own signal. Since shareholders pay \( f \), they must condition their voting strategy on the recommendation. (Otherwise, they could improve by using the same voting strategy, but saving fee \( f \).) To condition the voting strategy on the recommendation means either to vote yes after for and no after against, or the opposite voting strategy. In either case, no shareholder is pivotal since all vote in the same way after a given vote recommendation.
   A shareholder can improve by not paying \( f \) and voting, e.g. yes. Hence, there is no symmetric equilibrium with this information-acquisition strategy.

4. Consider now Subscribe-Invest, i.e., shareholders subscribe to the PA and invest in an own signal. Since shareholders pay both \( f \) and \( c \), they must condition their voting strategy on both the vote recommendation and their own signal. Otherwise, they could improve exhibiting the same voting behavior, but saving costs. This means that in fact only two voting strategies remain. Case 1: The strategy to vote yes except if the vote recommendation is against and the own signal is a, when \( i \) votes according to the own signal, as illustrated in Table A.2. In this case 1, no shareholder is pivotal if the PA recommends for (as the recommendation is common for all shareholders). Hence, shareholder \( i \) can only be pivotal if the recommendation is against. If so, \( i \) votes according to the own signal. Hence, deviating to strategy UNIS would not change the outcome because either \( i \) is not pivotal or \( i \) would also
vote according to the own signal. UNIS, however, saves fee $f$. Thus, the strategy profile of case 1 (Table A.2) cannot be a symmetric equilibrium.

Case 2: The strategy to vote no except if both the vote recommendation is for and the own signal is $b$ (for board), when $i$ votes according to the own signal, as in Table A.3. The analogous argument as above for case 1 applies, as follows: In this case 2, no shareholder is pivotal if the PA recommends against (as the recommendation is common for all shareholders). Hence, shareholder $i$ can only be pivotal if the vote recommendation is for. If so, $i$ votes according to the own signal. Hence, deviating to strategy UNIS would not change the outcome because either $i$ is not pivotal or $i$ would also vote according to the own signal. UNIS, however, saves fee $f$. Thus, the strategy profile of case 2 (Table A.3) cannot be a symmetric equilibrium.

Therefore, there cannot be a symmetric equilibrium with information-acquisition strategy Subscribe-Invest, in which shareholders unconditionally buy both the PA’s recommendation and an own signal.
2 Asymmetric Equilibria

The main text provides one key result for asymmetric equilibria (Proposition 3), while it summarizes the others. This section of the Supplementary Online Material (SOM) provides the detailed analysis for what happens when we drop the symmetry assumption. We first show that without a PA, the number of shareholders who invest in own research is bounded from above (SOM Section 2.1). We then show how admitting a PA alters this result. More specifically, we analyze asymmetric equilibria with a PA first by establishing three lemmata (SOM Section 2.2), then by comparing the shareholders’ research incentives with and without a PA (SOM Section 2.3), and finally by comparing the resulting decision quality (SOM Section 2.3). As we show, the number of shareholders who invest or conditionally invest, as well as the decision quality, weakly increase due to the presence of a PA, confirming the conclusions from the analysis of symmetric equilibria.

2.1 Asymmetric Equilibria in Benchmark Setting without a Proxy Advisor

Consider again the benchmark setting in which no PA is admitted. While Proposition 1 stated that under Assumption BIB, there is no symmetric equilibrium in which every shareholder invests in own research, the next result extends this to asymmetric equilibria in some parameter range. It also states that generally, in equilibrium without a PA there are always some shareholders not investing in research.

**Proposition 2.1 (ASYM without PA).** Suppose no PA is admitted.

i. If $\frac{\ell_B}{\ell_S} \geq \frac{N+1}{2}$, there does not exist an equilibrium in which any shareholder invests in own research. In any Pareto-efficient equilibrium $N' \in \{\frac{N+1}{2}, \ldots, N\}$ shareholders (i.e., a majority) play Rubber-stamping and $N - N'$ play Protest, which leads to decision quality $\Pi(\sigma_{\text{Rubber}}) = q_B$.

ii. Otherwise, i.e., if $\frac{\ell_B}{\ell_S} < \frac{N+1}{2}$, in equilibrium the number of shareholders who invest in own research is at most $z_1 := N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$. In the Pareto-efficient equilibrium under sufficiently small costs $c$, $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor = z_1$ shareholders play UNIS and $\lfloor \frac{\ell_B}{\ell_S} \rfloor$ shareholders play Rubber-stamping.

**Proof.** The proof is organized in four paragraphs.

**All Strategies.** Consider all pure strategies. First, those who do not buy the signal have the following pure strategies: voting yes (Rubber-stamping) and voting no (Protest).

Second, a shareholder that invests into the own signal must condition his voting behavior on the signal and be pivotal in at least one draw of nature. Otherwise, i.e., when voting unconditionally or never being pivotal, there is an improvement by keeping the voting strategy and not investing into the signal, saving costs $c$. A shareholder that invests and conditions on the signal can either vote in line with his signal (i.e., vote yes if $b$ (for board) and no if $a$

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[^54]: The mathematical expression $\lfloor z \rfloor$ is defined as the largest integer that is lower or equal to $z$. 

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(against board), which we call UNIS, or do the opposite \((yes \text{ iff } a)\). The opposite \((yes \text{ iff } a)\) cannot be part of an equilibrium strategy. Indeed, if voting \(no\) after receiving signal \(b\) (for board) is a best response, then, conditional on signal \(b\), state \(A\), which does not match the board’s proposal, must be more likely than \(B\), the state that matches the board’s proposal. Since the information technology of signals is monotonic, receiving signal \(a\) (against board) makes state \(A\) even more likely such that this voter also prefers to vote \(no\) if the signal is \(a\) (against). Hence, the opposite voting strategy can be ruled out and only UNIS remains for those who buy the signal.

**Conditions for Investing in Own Signal.** For a given strategy profile \(\sigma\), a given realization of signals, and a shareholder \(i\), let us define two numbers \(d\) and \(\delta_{-i}\). For the voters who have not invested, let \(d\) be the number of unconditional \(yes\)-votes minus the unconditional \(no\) votes; for the voters who have invested, let \(\delta_{-i}\) be the number of \(a\) (against) signals received minus the number of \(b\) (for board) signals received when excluding the focal shareholder \(i\). (Recall that in equilibrium those who received signal \(b\) will vote \(yes\) and those who received signal \(a\) will vote \(no\)).

Suppose shareholder \(i\) invested in research and is pivotal. Pivotality implies that the number of total votes of others, \(N - 1\), is fifty-fifty split in \(yes\) and \(no\) votes. This implies that \(d = \delta_{-i}\).

The two necessary and jointly sufficient conditions for optimality of voting according to the own signal are: \(\ell_B + 1\ell_S > \delta_{-i}\ell_S\), which is required for (and implies) optimality of voting \(yes\) when the own signal is \(b\), and \(\delta_{-i}\ell_S + 1\ell_S < \ell_B\), which is required for (and implies) optimality of voting \(no\) after the own signal \(a\). This yields \(d = \delta_{-i} \in \left(\frac{\ell_B}{\ell_S} - 1, \frac{\ell_B}{\ell_S} + 1\right)\). In fact, the interval is open. Suppose to the contrary, that \(d = \frac{\ell_B}{\ell_S} - 1\). Then \(\delta_{-i} = d\) implies \(\delta_{-i}\ell_S + 1\ell_S = \ell_B\), i.e., an informed shareholder \(i\) is indifferent between voting \(yes\) and voting \(no\) after receiving signal \(a\). After receiving signal \(b\), this shareholder prefers to vote \(yes\). Hence, if this shareholder would unconditionally vote \(yes\), she would induce the same decision quality. Therefore, this shareholder could unilaterally improve (upon her strategy UNIS) by Rubber-stamping, i.e., not investing into an own signal and voting \(yes\) unconditionally, which saves costs \(c > 0\). Analogously, it would be beneficial to switch to unconditionally voting \(no\) in case of \(d = \frac{\ell_S}{\ell_S} + 1\).

**Part i.** For part i. of the proposition, we use in particular that we have \(d = \delta_{-i} > \frac{\ell_B}{\ell_S} - 1\) when a shareholder invests, while \(\ell_S \geq \frac{N+1}{2}\) by assumption. Thus, \(d > \frac{N+1}{2} - 1\) and hence \(d \geq \frac{N+1}{2}\) (as \(d\) is a natural number). Hence, the assumption that a shareholder invests in own research leads to the implication that at least \(d \geq \frac{N+1}{2}\) more shareholders unconditionally vote \(yes\) than unconditionally vote \(no\). The latter, however, implies that no voter is ever pivotal (since there is always a majority voting \(yes\)). This, in turn, contradicts the assumption that a shareholder invests in research. Thus, there cannot be an informed voter in equilibrium.

Without informed voter, each shareholder votes either unconditionally \(yes\) (Rubber-stamping) or \(no\) (Protest). Information quality is \(q_B\) if a majority votes unconditionally \(yes\) and \(1 - q_B\) otherwise. Hence, in any Pareto-efficient equilibrium \(N^* \in \{\frac{N+1}{2}, \ldots, N\}\) play the strategy of Rubber-stamping and \(N - N^*\) play the strategy of Protest such that decision quality is \(\Pi^* = q_B\).
Part ii. In order to prove the upper bound for the number of informed shareholders, we use again that we have $d > \frac{\ell_B}{\ell_S} - 1$ when a shareholder invests. This implies $d \geq \lfloor \frac{\ell_B}{\ell_S} \rfloor$, as $d$ is a natural number. To create a vote difference of $d$ among those who did not invest, it takes at least $d$ voters, e.g., when $d$ vote yes while zero vote no. Hence, the number of informed voters is at most $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor = z_1$.

We now prove that $\lfloor \frac{\ell_B}{\ell_S} \rfloor$ players voting always yes (strategy Rubber-stamping) and $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ players voting according to their private signals (strategy UNIS) is indeed an equilibrium. To this end, we show that no shareholder in these two groups has an incentive to deviate from her strategy.

- Consider a shareholder who invests in an own signal and votes according to it (strategy UNIS). She is pivotal if the others’ votes are split, which happens if and only if there are $d = \lfloor \frac{\ell_B}{\ell_S} \rfloor = \delta - 1$ more $a$-signals than $b$-signals among the other informed shareholders. In that occasion, deviating to voting no after signal $b$ is suboptimal because it would reject the board’s proposal although $\ell_B + 1\ell_S \leq \delta - 1\ell_S$; and deviating to voting yes after signal $a$ is suboptimal because it would accept the board’s proposal although $\ell_B \geq \delta - 1\ell_S + 1\ell_S$. As pivotality occurs with positive probability, deviations certainly reduce decision quality, while they can only save costs $c$ that are by assumption arbitrarily small.

- Consider now a shareholder who always votes yes. Even if she knew her signal in case of being pivotal, she would never want to deviate to voting no, as she is pivotal if and only if there are $\frac{N-1}{2} - \lfloor \frac{\ell_B}{\ell_S} \rfloor - 1$ players who obtained positive signals and $\frac{N-1}{2}$ players with negative ones: A negative signal would make her prefer to vote no if and only if $\ell_B + (\frac{N-1}{2} - \lfloor \frac{\ell_B}{\ell_S} \rfloor) \cdot \ell_S < \frac{N-1}{2} \cdot \ell_S + \ell_S$, which is equivalent to $\ell_B - \lfloor \frac{\ell_B}{\ell_S} \rfloor \cdot \ell_S < 0$ and thus to $\frac{\ell_B}{\ell_S} < \lfloor \frac{\ell_B}{\ell_S} \rfloor$, which is a contradiction.

We will now prove that the equilibrium is Pareto-efficient. To this end, we first prove that for any equilibrium in which a positive number of players invests in own research, the difference $d$ between players always voting yes and those always voting no must equal $\lfloor \frac{\ell_B}{\ell_S} \rfloor$. Above, we have already derived that $d \in (\frac{\ell_B}{\ell_S} - 1, \frac{\ell_B}{\ell_S} + 1)$, which can only happen for $d = \lfloor \frac{\ell_B}{\ell_S} \rfloor$ or $d = \lceil \frac{\ell_B}{\ell_S} \rceil + 1$. For any fixed strategy profile and signal distribution, let $\delta$ denote the difference between the number of informed shareholders voting no and informed shareholders voting yes. A shareholder always voting yes will then be pivotal if and only if $d - 1 = \delta$ or, equivalently, $d = \delta + 1$. Now, let us assume that this shareholder still had the opportunity for own research and obtained a signal which happens to contradict the board’s proposal. This shareholder would like to vote no if $\ell_B < \delta \ell_S + \ell_S$, which is equivalent to $\ell_B < d\ell_S$ or $\frac{\ell_B}{\ell_S} < d$. Thus, whenever $d > \frac{\ell_B}{\ell_S}$ and costs are sufficiently low, we cannot have a rubber-stamping player in equilibrium because such a player would benefit from deviating to UNIS. As indeed $\lfloor \frac{\ell_B}{\ell_S} \rfloor + 1 > \frac{\ell_B}{\ell_S}$, $d = \lfloor \frac{\ell_B}{\ell_S} \rfloor + 1$ cannot result in an equilibrium. Thus, we overall must have $d = \lfloor \frac{\ell_B}{\ell_S} \rfloor$ in any equilibrium in which some shareholders invest in own research.

Overall, we now have established that apart from trivial equilibria in which no shareholder invests in private research, there can only be equilibria in which the difference between those always voting yes and those always voting no is exactly equal to $\lfloor \frac{\ell_B}{\ell_S} \rfloor$. Possible non-trivial equilibria are thus characterized by $\lfloor \frac{\ell_B}{\ell_S} \rfloor + \alpha$ shareholders always voting yes and $\alpha$ shareholders
always voting no, with α being a non-negative integer. Define \( \pi(l, k) := \sum_{i=k}^{l} \binom{l}{i} q_S^i (1-q_S)^{l-i} \), i.e., the probability that among \( l \) realizations of signals with precision \( q_S \) at least \( k \) are correct. Decision quality is then

\[
\Pi(\sigma) = q_B \cdot \pi \left( N - (d + \alpha), \frac{N + 1}{2} - d \right) + (1 - q_B) \cdot \pi \left( N - (d + \alpha), \frac{N + 1}{2} \right)
\]

for \( d = \lfloor \frac{\nu}{\ell_S} \rfloor \), as \( N - (d + \alpha) \) shareholders play strategy UNIS, \( d + \alpha \) play strategy Rubber-stamping, and \( \alpha \) play Protest. Since the function \( \pi \) is increasing in its first argument, decision quality is maximized for \( \alpha = 0 \). Hence, in the equilibrium of this type that provides the highest decision quality, \( N - \lfloor \frac{\nu}{\ell_S} \rfloor \) play strategy UNIS, while \( \lfloor \frac{\nu}{\ell_S} \rfloor \) play strategy Rubber-stamping. The corresponding decision quality is larger than the highest decision quality of all equilibria in which no shareholder invests in own research, which is \( q_B \). The reason is the following: whenever the proposal of the board is accepted, the decisions coincide, but when a proposal is rejected, we have at least \( d + 1 \) more signals against the proposal than for the proposal, which makes it more likely that the proposal is wrong: \( \ell_B \leq (d + 1)\ell_S \) for \( d = \lfloor \frac{\nu}{\ell_S} \rfloor \). \( \square \)

To understand SOM Proposition 2.1, consider a shareholder who invested in own research. This investment can only be part of an equilibrium if this shareholder conditions on her own signal in some instance in which she is pivotal. In particular, this shareholder must vote no if the signal is a (against). This is a best response if pivotality implies that a sufficient number of other informed shareholders also have received information against the board’s proposal. This, in turn, is possible in strategy profiles in which several uninformed shareholders Rubber-stamp the board’s proposal. When the number of shareholders who Rubber-stamp is by roughly \( \frac{\nu}{\ell_S} \) larger than the number of shareholders who vote unconditionally no (i.e., play Protest), then there might indeed be incentives to invest in own research and vote according to one’s signal. If this difference, however, exceeds half of all shareholders, as considered in part i., then it is impossible to be pivotal in the first place. Otherwise, i.e., in the case addressed in part ii., it is possible to have informed shareholders, but their number is bounded from above by \( N - \lfloor \frac{\nu}{\ell_S} \rfloor \). It turns out that the strategy profile with the highest decision quality is then \( \sigma^\mu \nu \), with \( \mu = N - \lfloor \frac{\nu}{\ell_S} \rfloor \) shareholders investing in own research and voting according to signal (strategy UNIS), and \( \nu = \lfloor \frac{\nu}{\ell_S} \rfloor \) shareholders playing strategy Rubber-stamping. This strategy profile is Pareto-efficient and yields the upper bound for the decision quality. For the description of decision quality it is helpful to define \( \pi(l, k) \) as the probability that among \( l \) realizations of signals with precision \( q_S \) at least \( k \) are correct, i.e., \( \pi(l, k) := \sum_{i=k}^{l} \binom{l}{i} q_S^i (1-q_S)^{l-i} \). Then the decision quality in case of part ii. of SOM Proposition 2.1 is \( \Pi(\sigma) = q_B \cdot \pi(z_1, z_1 - \frac{N-1}{2}) + (1 - q_B) \cdot \pi(z_1, \frac{N+1}{2}) \), where still \( z_1 := N - \lfloor \frac{\nu}{\ell_S} \rfloor \).

Comparative statics imply that the maximal number of shareholders who invest is decreasing in the board’s relative information quality \( \frac{\nu}{\ell_S} \), starting with \( N - 1 \) for \( \lfloor \frac{\nu}{\ell_S} \rfloor = 1 \), decreasing down to \( \frac{N+1}{2} \) for \( \lfloor \frac{\nu}{\ell_S} \rfloor = \frac{N-1}{2} \), and then discontinuously jumping to 0.\(^{55}\) This

\(^{55}\)Only if \( \frac{\nu}{\ell_S} < 1 \), which is the negation of Assumption BIB, all \( N \) shareholders could be informed.
validates the insight we gained from the symmetric equilibria: Without a PA, well informed boards reduce shareholders’ research incentives.\footnote{Their overall effect on decision quality might however still be positive -- we will discuss comparative-static effects on decision quality further below in SOM Section 2.4.}

### 2.2 Lemmata for Asymmetric Equilibria with a Proxy Advisor

Before analyzing the effects of introducing a PA on shareholders’ research incentives (SOM Section 2.3) and on decision quality (SOM Section 2.3), we establish three lemmata about asymmetric equilibria when a PA is admitted. SOM Lemma 2.1 shows two properties of equilibrium behavior, SOM Lemma 2.2 provides bounds on the number of informed shareholders, and SOM Lemma 2.3 characterizes the Pareto-efficient strategy profiles.

**Lemma 2.1 (ASYM with PA: Equilibrium Behavior).** Let Assumption PAF hold. Let costs $c > 0$ be arbitrarily small and let fee $f > 0$ be sufficiently smaller. In equilibrium the following statements hold:

i. There is no shareholder who buys the vote recommendation and unconditionally invests in own research, i.e., uses Subscribe-Invest.

ii. Every shareholder who acquires an own signal votes according to this signal, i.e., votes yes if the signal is $b$ (board) and vote no if the signal is $a$ (against).

**Proof.** We first show part i. and then use part i. to show part ii.

i. Suppose shareholder $i$ uses Subscribe-Invest. Since $i$ pays both $f$ and $c$ she must condition her voting strategy on both the PA’s vote recommendation and the own signal. This excludes unconditional voting (such as always yes) and voting conditional only on one type of information (such as only voting according to the vote recommendation, or only voting according to the signal). Indeed, compared to these strategies, the shareholder could improve by exhibiting the same voting behavior, but saving costs. This means that only one type of voting strategy remains, namely conditioning on both the vote recommendation and the own signal. Consider one such strategy, namely voting yes except if the vote recommendation is against and the signal is $a$, then the shareholder votes no (as in Table A.2). In this case, the shareholder votes yes after the for recommendation, independently of the own signal. She could improve by keeping the same voting behavior, but changing the information-acquisition strategy, switching to Subscribe-InvestIFF against, which saves costs $c$ with positive probability. Likewise, there is an improvement for all strategies of this type. In particular, consider voting no except if the vote recommendation is for and the own signal is $b$ (for board), as in Table A.3. Here, there is an improvement to Subscribe-InvestIFF for.

ii. There are four information-acquisition strategies that involve investing in an own signal: NotSubscribe-Invest, Subscribe-Invest, Subscribe-InvestIFF for, and Subscribe-InvestIFF against.

Suppose there is a shareholder $i$ who plays NotSubscribe-Invest and violates the assertion. This shareholder either votes independently of the own signal or votes for the opposite of
what the signal indicates. In the former case, \( i \) could improve by using the same voting strategy, but not acquiring a signal. That would lead to the same decision quality but reduce her costs. In the latter case, the shareholder is either never pivotal and could again improve by not acquiring the signal, or she is pivotal with positive probability. If pivotal with positive probability, \( i \) would only find it optimal to vote for the opposite of what the signal indicates if, conditional on signal \( b \) and pivotality, state \( A \) that does not match the board’s proposal is at least as likely as state \( B \) that does (and conditional on signal \( a \) and pivotality, state \( B \) is at least as likely as \( A \)). However, as the signal is drawn independently of the occurrence of pivotality and is informative as \( q_s > 0.5 \), this is impossible (conditional on signal \( b \), state \( B \) is strictly more likely than it would be when conditioning on signal \( a \)). Therefore, a shareholder who uses NotSubscribe-Invest in equilibrium plays strategy UNIS.

There is no shareholder who plays Subscribe-Invest, as shown in part i. of this proof.

Suppose there is a shareholder \( i \) who plays Subscribe-InvestIFF for and violates the assertion. After vote recommendation for, this shareholder either votes independently of the signal or for the opposite of what the signal indicates. There are improvements by switching to not acquiring the signal or to voting according to the signal, in complete analogy to the information-acquisition strategy NotSubscribe-Invest above. Likewise, this holds for Subscribe-InvestIFF against.

\[ \textbf{Lemma 2.2 (ASYM with PA: Bounds).} \text{ Let Assumption PAF hold. Let costs } c > 0 \text{ be arbitrarily small and let fee } f > 0 \text{ be sufficiently smaller. In equilibrium the following holds.} \]

i. If \( \frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2} \), then the number of shareholders who invest in own research is at most \( N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor \) after vote recommendation for and at most \( N - \left\lfloor \frac{|B-P|}{\ell_S} \right\rfloor \) after vote recommendation against.

ii. If \( \frac{|B-P|}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S} \), then no shareholder invests in own research after vote recommendation for and at most \( N - \left\lfloor \frac{|B-P|}{\ell_S} \right\rfloor \) shareholders invest after vote recommendation against.

iii. Otherwise, i.e., if \( \frac{N+1}{2} \leq \frac{|B-P|}{\ell_S} \), no shareholder invests in own research.

\[ \text{Proof.} \text{ We first establish two statements and then proceed with the three parts of SOM Lemma 2.2.} \]

(B1) In equilibrium, the number of shareholders who invest in own research is at most \( \max\{N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor, 0\} \) after vote recommendation for.

(B2) In equilibrium, the number of shareholders who invest in own research is at most \( \max\{N - \left\lfloor \frac{|B-P|}{\ell_S} \right\rfloor, 0\} \) after vote recommendation against. This holds for both (i) \( \ell_B \geq \ell_P \) and (ii) \( \ell_B < \ell_P \).

Ad B1 Suppose statement (B1) is violated. Then there is an equilibrium where the number of informed shareholders after vote recommendation for, \( x_1 \), satisfies \( x_1 > N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor \) and \( x_1 > 0 \).
Since the information-acquisition strategy Subscribe-Invest is never part of an equilibrium (by part i. of SOM Lemma 2.1), \( x_1 > 0 \) implies that every informed player after vote recommendation \( for \) either plays NotSubscribe-Invest or Subscribe-InvestIFF \( for \).

Suppose first that there is a shareholder \( i \) who plays Subscribe-InvestIFF \( for \) in equilibrium. This player must be pivotal after vote recommendation \( for \) with positive probability, otherwise she could improve by saving costs \( c \) without affecting decision quality. Pivotality implies that \( \frac{N-1}{2} \) of the \( N-1 \) other shareholders vote \( yes \) and \( \frac{N-1}{2} \) vote \( no \). Let \( \delta \) be the difference between the \( yes \)-votes and \( no \)-votes of uninformed players (i.e., those who have not acquired a signal) after vote recommendation \( for \). By part ii. of SOM Lemma 2.1, all shareholders who have acquired a signal after vote recommendation \( for \) must vote according to it. Therefore, conditional on having received recommendation \( for \) and on pivotality, shareholder \( i \) knows that among the other informed shareholders, exactly \( \delta_{-i} = d \) more have received signal \( a \) than \( b \).

(Again \( \delta_{-i} \) designates the number of \( a \) (against) signals received minus the number of \( b \) (for board) signals received when excluding the focal shareholder \( i \).) However, as the number of uninformed players \( y_i := N - x_1 \) satisfies \( y_i < \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor \), the signal difference that makes player \( i \) pivotal satisfies \( \delta_{-i} = d \leq y_i < \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor \). As \( \delta_{-i} \) is a natural number, \( \delta_{-i} < \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor \) implies \( \delta_{-i} \geq \frac{\ell_B + \ell_P}{\ell_S} - 1 \), which is equivalent to \( \ell_B + \ell_P \geq \delta_{-i}\ell_S + 1\ell_S \). The final inequality means that state \( B \), which matches the board’s proposal, is more likely than state \( A \), which does not match the proposal, when \( i \) is pivotal after vote recommendation \( for \), even if \( i \) has received signal \( a \). Hence, shareholder \( i \) could improve by voting \( yes \) after vote recommendation \( for \) and not investing in the own signal, which would weakly increase decision quality and strictly decrease costs, contradicting the assumption that the strategy profile with \( i \) playing Subscribe-InvestIFF \( for \) was an equilibrium.

Suppose now that there is no shareholder who plays Subscribe-InvestIFF \( for \) in equilibrium. Then all informed shareholders play NotSubscribe-Invest. By part ii. of SOM Lemma 2.1, all of them play UNIS. Consider the unilateral deviation of a shareholder \( i \) from UNIS to strategy CAIS. By the assumption on costs that \( f \) is sufficiently lower than \( c \), this saves costs. Moreover, this would weakly improve decision quality: after the \( against \) recommendation the decision is the same, but after the \( for \) recommendation the decision quality weakly improves. The argument for the latter claim is exactly as before when considering a player using Subscribe-InvestIFF \( for \). When shareholder \( i \) is pivotal after vote recommendation \( for \), we have \( \delta_{-i} = d \leq y_i = N - x_1 < \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor \), which implies \( \ell_B + \ell_P \geq \delta_{-i}\ell_S + 1\ell_S \), meaning that state \( B \) is more likely than state \( A \) even if \( i \) has received signal \( a \). Hence, the deviation to strategy CAIS is an improvement for shareholder \( i \), contradicting the assumption that the strategy profile with no shareholder playing Subscribe-InvestIFF \( for \) was an equilibrium.

Hence, if (B1) is violated, we have that no shareholder plays Subscribe-InvestIFF \( for \) in equilibrium and that a strategy profile without a shareholder playing Subscribe-InvestIFF \( for \) cannot be an equilibrium -- a contradiction.
Ad B2(i) Suppose statement (B2) is violated and \( \ell_B \geq \ell_P \) holds. Then there is an equilibrium where the number of informed shareholders after the \textit{against} recommendation, \( x_2 \), satisfies \( x_2 > N - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor \) and \( x_2 > 0 \).

Since the information-acquisition strategy Subscribe-Invest is never part of an equilibrium (by part i. of SOM Lemma 2.1), \( x_2 > 0 \) implies that every informed player after the \textit{against} recommendation either plays NotSubscribe-Invest or Subscribe-InvestIFF\textit{against}.

Suppose first that there is a shareholder \( i \) who plays Subscribe-InvestIFF\textit{against} (e.g., strategy CAIS) in equilibrium. This shareholder must be pivotal with positive probability, otherwise she could improve by saving costs without affecting the decision quality. By part ii. of SOM Lemma 2.1, all shareholders who are informed after vote recommendation \textit{against} vote according to their signal. In particular, shareholder \( i \) must vote \textit{no} after signal \( a \), which requires: \( \ell_B < \ell_P + \delta_{-i} \ell_S + \ell_S \), where \( \delta_{-i} \) is again the vote difference of the other informed shareholders. (A weak inequality cannot be part of an equilibrium strategy as the deviation to voting always \textit{yes} after vote recommendation \textit{against} and saving costs \( c \) would be an improvement.) That is, \( \delta_{-i} > \frac{\ell_B - \ell_P}{\ell_S} - 1 \). Since all informed shareholders vote according to their signal, the condition above and pivotality imply that at least \( \frac{\ell_B - \ell_P}{\ell_S} \) shareholders are uninformed, which is in contradiction to the assumption \( x_2 > N - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor \) as this inequality implies that strictly less than \( \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor \) are uninformed.\(^{57}\) Hence, in equilibrium no shareholder can play Subscribe-InvestIFF\textit{against}.

Suppose now that there is no player who plays Subscribe-InvestIFF\textit{against} in equilibrium. Then all informed shareholders after the \textit{against} recommendation play strategy UNIS (by part ii. of SOM Lemma 2.1). Consider the unilateral deviation of a shareholder \( i \) from UNIS to Subscribe-InvestIFF\textit{for}, voting unconditionally \textit{yes} after vote recommendation \textit{against}. By the assumption on costs that \( f \) is sufficiently lower than \( c \), this saves costs. Moreover, this would weakly improve decision quality because after the \textit{for} recommendation, the decision is the same, but after the \textit{against} recommendation, the decision weakly improves, as we show now. For the given strategy profile in the assumed equilibrium, we define the following numbers for the case that the vote recommendation is \textit{against}: Let \( y_2 = N - x_2 \) be the number of uninformed shareholders, let \( d \) be the difference of \textit{yes}-votes and \textit{no}-votes among the \( y_2 \) uninformed shareholders, and let \( \delta_{-i} \) be the difference of \( a \)-signals and \( b \)-signals of the \( x_2 \) informed shareholders. When shareholder \( i \) is pivotal after vote recommendation \textit{against}, we have \( \delta_{-i} = d \leq y_2 = N - x_2 < \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor \). For \( \ell_B \geq \ell_P \), and since \( \delta_{-i} \) is a natural number, this implies \( \delta_{-i} \leq \frac{\ell_B - \ell_P}{\ell_S} - 1 \), which is equivalent to \( \ell_B \geq \ell_P + \delta_{-i} \ell_S + 1 \ell_S \). The final inequality means that state \( B \), which matches the board’s proposal, is more likely than state \( A \), which does not match the proposal, when shareholder \( i \) is pivotal after vote recommendation \textit{against}, even if \( i \) has received signal \( a \). Hence, the deviation weakly improves decision quality, while it strictly saves costs, contradicting the assumption that the strategy profile with no

\(^{57}\)For an integer \( x, x \geq \lceil \tau \rceil \) is equivalent to \( x > \tau - 1 \).
shareholder playing Subscribe-InvestIFF against was an equilibrium.

Hence, if B2(i) is violated, we have that no shareholder plays Subscribe-InvestIFF against in equilibrium and that a strategy profile without a shareholder playing Subscribe-InvestIFF cannot be an equilibrium -- a contradiction.

Ad B2(ii) Suppose statement (B2) is violated and $\ell_B < \ell_P$ holds. Then there is an equilibrium where the number of informed shareholders after the against recommendation, $x_2$, satisfies $x_2 > N - \left\lfloor \frac{[\ell_B - \ell_P]}{\ell_S} \right\rfloor$ and $x_2 > 0$.

Since the information-acquisition strategy Subscribe-Invest is never part of an equilibrium (by part i. of SOM Lemma 2.1), $x_2 > 0$ implies that every informed player after the against recommendation either plays NotSubscribe-Invest or Subscribe-InvestIFF.

Suppose first that there is a shareholder $i$ who plays Subscribe-InvestIFF against (e.g., strategy CAIS) in equilibrium. For the given strategy profile in the assumed equilibrium, we define the following numbers for the case that the vote recommendation is against: Let $y_2 = N - x_2$ be the number of uninformed shareholders, let $\tilde{d} = -d$ be the difference of no-votes and yes-votes among the $y_2$ uninformed shareholders, and let $\tilde{\delta}_{-i} = -\delta_{-i}$ be the difference of $b$-signals and $a$-signals (in favor of the proposal) of the $x_2$ informed shareholders who are not $i$. When shareholder $i$ is pivotal after vote recommendation against, we have $\tilde{\delta}_{-i} = \tilde{d} \leq y_2 = N - x_2 < \left\lfloor \frac{[\ell_B - \ell_P]}{\ell_S} \right\rfloor$. For $\ell_B < \ell_P$ and since $\tilde{\delta}_{-i}$ is a natural number, this implies $\tilde{\delta}_{-i} \leq \frac{\ell_B - \ell_P}{\ell_S} - 1$, which is equivalent to $\ell_B + \tilde{\delta}_{-i} \ell_S + 1 \ell_S \leq \ell_P$. The final inequality means that state $B$, which matches the board’s proposal, is less likely than state $A$, which does not match the proposal, when shareholder $i$ is pivotal after vote recommendation against, even if $i$ has received signal $b$. Hence, $i$ could improve by deviating to unconditionally voting no after vote recommendation against and not investing in an own signal. This deviation weakly improves decision quality, while it strictly saves costs, contradicting the assumption that the strategy profile with $i$ playing Subscribe-InvestIFF against was an equilibrium.

Suppose now that there is no player who plays Subscribe-InvestIFF against. Then all informed shareholders after the against recommendation play UNIS. Consider the unilateral deviation of a shareholder $i$ from UNIS to Subscribe-InvestIFF for, unconditionally voting no after vote recommendation against. By the assumption on costs that $f$ is sufficiently lower than $c$, this saves costs. Moreover, this would weakly improve decision quality by the same argument used immediately above: When shareholder $i$ is pivotal after vote recommendation against, we have $\tilde{\delta}_{-i} = \tilde{d} \leq y_2 = N - x_2 < \left\lfloor \frac{[\ell_B - \ell_P]}{\ell_S} \right\rfloor$, which implies $\ell_B + \tilde{\delta}_{-i} \ell_S + 1 \ell_S \leq \ell_P$. The final inequality means that state $B$ is less likely than state $A$ when shareholder $i$ is pivotal after vote recommendation against even if $i$ has received signal $b$.

Hence, if B2(ii) is violated, we have that no shareholder plays Subscribe-InvestIFF against in equilibrium and that a strategy profile without a shareholder playing Subscribe-InvestIFF against cannot be an equilibrium -- a contradiction.
Now, we can use (B1) and (B2) to address the three parts of SOM Lemma 2.2.

i. By the assumption $\frac{\ell_B + \ell_P}{\ell_s} < \frac{N+1}{2}$, we have $\max\{N - \lfloor \frac{\ell_B + \ell_P}{\ell_s} \rfloor, 0\} = N - \lfloor \frac{\ell_B + \ell_P}{\ell_s} \rfloor$ and the result after vote recommendation for follows directly from statement (B1). Likewise, we have $\lfloor \frac{\ell_B - \ell_P}{\ell_s} \rfloor \leq \frac{\ell_B + \ell_P}{\ell_s} < \frac{N+1}{2}$, implying $\max\{N - \lfloor \frac{\ell_B - \ell_P}{\ell_s} \rfloor, 0\} = N - \lfloor \frac{\ell_B - \ell_P}{\ell_s} \rfloor$ and the result after vote recommendation against follows directly from statement (B2).

ii. By the assumption $\lfloor \frac{\ell_B - \ell_P}{\ell_s} \rfloor < \frac{N+1}{2}$, we have $\max\{N - \lfloor \frac{\ell_B - \ell_P}{\ell_s} \rfloor, 0\} = N - \lfloor \frac{\ell_B - \ell_P}{\ell_s} \rfloor$ and the result after vote recommendation against follows directly from statement (B2).

It remains to show that after vote recommendation for, no shareholder invests in own research. Suppose, in contrast, that there is an equilibrium where the number $x_1$ of informed shareholders after vote recommendation for satisfies $x_1 > 0$.

Statement (B1) implies $x_1 \leq N - \lfloor \frac{\ell_B + \ell_P}{\ell_s} \rfloor$. Hence, the number of uninformed shareholders after vote recommendation for satisfies $y_1 = N - x_1 \geq \lfloor \frac{\ell_B + \ell_P}{\ell_s} \rfloor$, which implies $y_1 > \frac{\ell_B + \ell_P}{\ell_s} - 1$. The assumption $\frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_s}$ now implies $\frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_s} < y_1 + 1$. Hence, there are at least $\frac{N+1}{2}$ uninformed shareholders after vote recommendation for. Therefore, no shareholder can be pivotal in that case. Any of the $x_1 > 0$ shareholders who invest in an own signal can beneficially deviate to (still) buying the vote recommendation, but not investing in an own signal after vote recommendation for, as this saves costs and does not affect decision quality.

iii. Suppose, in contrast, that there is an equilibrium where either (1) the number of informed shareholders after the for recommendation, $x_1$, satisfies $x_1 > 0$, or (2) the number of informed shareholders after the against recommendation, $x_2$, satisfies $x_2 > 0$, or both. We show that there is a contradiction, first for statement (1) and then for statement (2).

The first case (1) is excluded by statement B1, exactly as shown immediately above.

In the second case (2), statement B2 implies $x_2 \leq N - \lfloor \frac{\ell_B - \ell_P}{\ell_s} \rfloor$. Hence, the number of uninformed shareholders after vote recommendation against satisfies $y_2 = N - x_2 \geq \lfloor \frac{\ell_B - \ell_P}{\ell_s} \rfloor$, which implies $y_2 > \frac{\ell_B - \ell_P}{\ell_s} - 1$. The assumption $\frac{N+1}{2} \leq \frac{\ell_B - \ell_P}{\ell_s}$ now implies $\frac{N+1}{2} \leq \frac{\ell_B - \ell_P}{\ell_s} < y_2 + 1$. Hence, there are at least $\frac{N+1}{2}$ uninformed shareholders after vote recommendation against. Therefore, no shareholder can be pivotal in that case. Any of the $x_2 > 0$ shareholders who invest in an own signal can beneficially deviate to (still) buying the vote recommendation, but not investing in an own signal after vote recommendation against, as this saves costs and does not affect decision quality.

\[\square\]

**Lemma 2.3 (ASYM with PA: Pareto-efficient Equilibria).** Let Assumption PAF hold. Let costs $c > 0$ be arbitrarily small and let fee $f > 0$ be sufficiently smaller. Then the following strategy profiles are Pareto-efficient equilibria.\(^{58}\)

\(^{58}\)We are particularly thankful to Stefan Kloessner who foresaw this result during a walk in the forest.
i. If $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B \geq \ell_P$, then $\left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor$ shareholders play strategy Rubber-stamping, $\left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor$ play strategy CAIS, and $N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor$ play strategy UNIS. Hence, decision quality is $\Pi^* = q_B q_B \pi(N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor, \frac{N+1}{2} - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor) + (1 - q_B)(1 - q_P)\pi(N - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor, \frac{N+1}{2}) + (1 - q_B)q_B\pi(N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor, \frac{N+1}{2}).$

ii. If $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B < \ell_P$, then $\left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor$ shareholders play Follow (buy recommendation and follow it), $\left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor$ play strategy CAIS, and $N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor$ play strategy UNIS. Hence, decision quality is $\Pi^* = q_B q_B \pi(N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor, \frac{N+1}{2} - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor) + (1 - q_B)(1 - q_P)\pi(N - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor, \frac{N+1}{2}) + q_B(1 - q_P)\pi(N - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor, \frac{N+1}{2}) + (1 - q_B)q_B\pi(N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor, \frac{N+1}{2}).$

iii. If $\frac{\ell_B - \ell_P}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S}$ and $\ell_B \geq \ell_P$, then $\left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor$ shareholders play strategy Rubber-stamping and $N - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor$ play strategy CAIS. Hence, decision quality is $\Pi^* = q_B q_B + q_B(1 - q_P)\pi(N - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor, \frac{N+1}{2} - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor) + (1 - q_B)q_B\pi(N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor, \frac{N+1}{2}).$

iv. If $\frac{\ell_B - \ell_P}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S}$ and $\ell_B < \ell_P$, then $\left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor$ shareholders play strategy Protest and $N - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor$ play strategy CAIS. Hence, decision quality is $\Pi^* = q_B q_B + (1 - q_B)q_B\pi(N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor, \frac{N+1}{2} - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor) + (1 - q_B)q_B\pi(N - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor, \frac{N+1}{2}).$

v. If $\frac{\ell_B - \ell_P}{\ell_S} \geq \frac{N+1}{2}$ and $\ell_B \geq \ell_P$, then $N' \in \{\frac{N+1}{2}, ..., N\}$ shareholders play strategy Rubber-stamping and $N - N'$ play strategy Protest. Hence, decision quality is $\Pi^* = q_B.$

vi. If $\frac{\ell_B - \ell_P}{\ell_S} \geq \frac{N+1}{2}$ and $\ell_B < \ell_P$, then $\frac{N+1}{2}$ shareholder play strategy Rubber-stamping, $\frac{N+1}{2}$ play Protest, and 1 plays Follow. Hence, decision quality is $\Pi^* = q_B.$

SOM Lemma 2.3 is illustrated in the lower panel of SOM Figure 2.1, where any region of the figure corresponds to one case of the lemma. In each region a Pareto-efficient equilibrium is indicated. The comparison with the Pareto-efficient equilibria in the benchmark setting without proxy advisor, illustrated in the upper lower panel of SOM Figure 2.1, is discussed in the next two subsections. The remainder of this subsection is dedicated to prove SOM Lemma 2.3.

**Proof.** We first determine the theoretically maximal decision quality. Then we show that in every area of the parameter space the claimed strategy profile reaches this maximum, is an equilibrium, and is Pareto-efficient.

Generally, decision quality is maximal for a given number of signals if for any realization of these signals, the alternative that is more likely to match the true state is implemented. Moreover, additional signals increase the maximal decision quality. In our model, we always have the signal of the board of quality $q_B$ and the signal of the PA of quality $q_P.$ Additionally, we can have a number of signals of quality $q_S.$ This number is restricted by SOM Lemma 2.2 and depends on whether the PA’s signal agrees with the board (i.e., the vote recommendation is for) or not (i.e., the vote recommendation is against). Let $\left\lfloor \tilde{n}_1 \right\rfloor$ denote the maximal number of signals of quality $q_S$ when the PA’s signal agrees with the board’s (vote recommendation for) and let $\left\lfloor \tilde{n}_2 \right\rfloor$ denote the maximal number of signals of quality $q_S$ when it disagrees (vote
Figure 2.1: Pareto-efficient strategy profiles. Upper panel without a PA based on SOM Proposition 2.1, lower panel with a PA based on SOM Lemma 2.3.
recommendation against). For any realization of signals of quality $q_S$ let $\delta$ be the number of $a$-signals minus the number of $b$-signals, i.e., the difference against the board, and define $\delta = 0$ if there is no such signal.

Then maximal decision quality under the constraints given by the upper bounds $\bar{n}_1$ and $\bar{n}_2$ is reached if and only if the following two conditions are satisfied:

(C1) When the PA’s signal is aligned with the board’s, then $\bar{n}_1$ signals of quality $q_S$ are generated and the proposal is accepted if $\delta < \frac{\ell_B + \ell_P}{\ell_S}$ and rejected if $\delta > \frac{\ell_B + \ell_P}{\ell_S}$ (and either accepted or rejected if $\delta = \frac{\ell_B + \ell_P}{\ell_S}$).

(C2) When the PA’s signal is against board, then $\bar{n}_2$ additional signals are generated and the proposal is accepted if $\delta < \frac{\ell_B - \ell_P}{\ell_S}$ and rejected if $\delta > \frac{\ell_B - \ell_P}{\ell_S}$ (and either accepted or rejected if $\delta = \frac{\ell_B - \ell_P}{\ell_S}$).

Indeed, when the PA’s signal agrees with the board’s and $\bar{n}_1 = 0$, the proposal of the board must be accepted. When the PA’s signal disagrees with the board’s and $\bar{n}_2 = 0$, the proposal must be accepted if $q_B > q_P$ and rejected if $q_B < q_P$. When the PA’s signal agrees with the board’s and $\bar{n}_1 > 0$, then the condition for acceptance in C1 is equivalent to the following.

$$\delta < \frac{\ell_B + \ell_P}{\ell_S}$$

$$\delta \ell_S < \ell_B + \ell_P$$

$$\delta \log \left( \frac{q_S}{1-q_S} \right) < \log \left( \frac{q_B}{1-q_B} \right) + \log \left( \frac{q_P}{1-q_P} \right)$$

$$\left( \frac{q_S}{1-q_S} \right)^{\delta} < \left( \frac{q_B}{1-q_B} \right) \times \left( \frac{q_P}{1-q_P} \right)$$

which is indeed the condition for state $B$ being more likely than state $A$, conditional on the signal realizations. The analogous arguments hold for $\delta > \frac{\ell_B + \ell_P}{\ell_S}$ and Condition C2. Observe that in C2, $\delta < \frac{\ell_B - \ell_P}{\ell_S}$ is negative when $q_B < q_P$, requiring more $b$-signals than $a$-signals to accept the proposal, as it should be. In the knife-edge case where $\delta = \frac{\ell_B - \ell_P}{\ell_S}$, respectively $\delta = \frac{\ell_B + \ell_P}{\ell_S}$, both states are equally likely, conditional on the signal realizations, and hence it does not matter for decision quality which decision is made. It is an important part of the Conditions C1 and C2 that the maximal possible number of signals of quality $q_S$ is generated, as any lower number of signals would lead to a strictly lower decision quality even if the decisions, given the realizations of signals, were according to the inequalities in C1 and C2.

We now set out to show for each case of SOM Lemma 2.3 that the claimed strategy satisfies the two conditions C1 and C2, with upper bounds $\bar{n}_1$ and $\bar{n}_2$ derived from SOM Lemma 2.2, and hence the maximum decision quality is attained. Moreover, we show that no shareholder has an incentive to deviate.

i. For $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B \geq \ell_P$, part i. of SOM Lemma 2.2 applies. Hence, $\bar{n}_1 = N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor$ and $\bar{n}_2 = N - \left\lfloor \frac{\ell_B - \ell_P}{\ell_S} \right\rfloor$. With the proposed strategy profile, we have $N - \left\lfloor \frac{\ell_B + \ell_P}{\ell_S} \right\rfloor$ shareholders who play strategy UNIS, which makes $\bar{n}_1$ informed shareholders after the $f$or recommendation. After the $against$ recommendation, we have additional
While the bounds of SOM Lemma 2.2 hold in any equilibrium, decision quality after deviations from a strategy profile is not per se. Hence, \( \delta > \frac{[t_{n-\ell}]}{t_s} \) is odd, \( \delta < \frac{[t_{n-\ell}]}{t_s} \) is even. Thus, for any realization of signals we have \( |\delta - [\frac{[t_{n-\ell}]}{t_s}]| \geq 1 \) because exactly one of these integers is even and one of them is odd. Hence, Condition C1 is satisfied.

After the against recommendation, the \( \frac{[t_{n-\ell}]}{t_s} \) shareholders vote yes and all other shareholders (those who play strategy CAIS and those who play strategy UNIS) invest in an own signal and vote according to it. Thus, the proposal is accepted if and only if \( \delta < [\frac{[t_{n-\ell}]}{t_s}] \). If \( [\frac{[t_{n-\ell}]}{t_s}] \) is even, then \( \bar{n}_2 \) is odd and thus \( \delta \) is odd. If \( [\frac{[t_{n-\ell}]}{t_s}] \) is odd, then \( \bar{n}_2 \) is even and thus \( \delta \) is even. Thus, for any realization of signals we have \( |\delta - [\frac{[t_{n-\ell}]}{t_s}]| \geq 1 \) because exactly one of these integers is even and one of them is odd. Hence, Condition C2 is satisfied.

Suppose the recommendation is for. Then, \( \frac{[t_{n+\ell}]}{t_s} \) shareholders vote yes (those playing strategy Rubber-stamping plus those playing strategy CAIS), while \( N - \frac{[t_{n+\ell}]}{t_s} = \bar{n}_1 \) shareholders invest in a signal of quality \( q_s \) and vote according to it. Thus, the proposal is accepted if and only if \( \delta < [\frac{[t_{n+\ell}]}{t_s}] \). Of the integers \( [\frac{[t_{n+\ell}]}{t_s}] \) and \( \delta \) exactly one is even and one is odd. If \( \delta < [\frac{[t_{n+\ell}]}{t_s}] \), then \( \delta < [\frac{[t_{n+\ell}]}{t_s}] \) and the proposal is accepted, as required by C1. If \( \delta > [\frac{[t_{n+\ell}]}{t_s}] \), then \( \delta > [\frac{[t_{n+\ell}]}{t_s}] \) and the proposal is rejected, as required by C1. Hence, Condition C1 is satisfied.

Hence, the proposed strategy profile satisfies Conditions C1 and C2. Thus, decision quality is maximized under the constraints that the maximal numbers of informed shareholders after recommendations for and against must not exceed upper bounds \( \bar{n}_1 \) and \( \bar{n}_2 \), respectively. Deviating to a strategy that lowers decision quality is not an improvement as costs are assumed to be sufficiently small and hence cannot compensate any loss in decision quality. Potentially beneficial deviations must therefore either reduce costs without affecting decision quality, or increase decision quality (which requires relaxing at least one of the constraints that impose upper bounds \( \bar{n}_1 \) and \( \bar{n}_2 \)).

The shareholders who Rubber-stamp cannot reduce costs, as their costs are zero. Suppose shareholder \( i \) deviates from strategy Rubber-stamping and improves decision quality. She is pivotal after the for recommendation if the others’ votes are split (there are \( N+\frac{1}{2} \) yes-votes and \( N+\frac{1}{2} \) no-votes). As \( [\frac{[t_{n+\ell}]}{t_s}] - 1 \) others play strategy Rubber-stamping and \( [\frac{t_{n+\ell}}{t_s}] - [\frac{[t_{n-\ell}]}{t_s}] \) play strategy CAIS, there are \( [\frac{[t_{n+\ell}]}{t_s}] - 1 \) unconditional yes-votes after recommendation for. The remaining \( N - [\frac{[t_{n+\ell}]}{t_s}] \) shareholders play UNIS and are thus informed. Hence, \( i \) is pivotal after for if and only if \( \delta = [\frac{[t_{n+\ell}]}{t_s}] - 1 \) (where \( \delta \) is again

\[ 59 \text{ Hence, } \delta = [\frac{[t_{n-\ell}]}{t_s}] \text{ is not possible.} \]

\[ 60 \text{ Hence, } \delta = [\frac{[t_{n+\ell}]}{t_s}] \text{ is not possible.} \]

\[ 61 \text{ We have to check for deviations that might potentially increase decision quality, although we have already established that decision quality in equilibrium is maximal under the constraints given by SOM Lemma 2.2. While the bounds of SOM Lemma 2.2 hold in any equilibrium, decision quality after deviations from a strategy profile is not per se restricted by the respective upper bound in SOM Lemma 2.2.} \]
the signal difference against the board among the informed). With Rubber-stamping, \( i \) would vote yes. To change the decision with a deviation, she must deviate to voting no. This deviation would be most likely to increase decision quality if she had received signal \( a \). Decision quality would improve in that case if \( \ell_B + \ell_P < (\lceil \frac{\ell_B - \ell_P}{\ell_S} \rceil - 1)\ell_S + 1\ell_S \), which is equivalent to \( \frac{\ell_B - \ell_P}{\ell_S} < \lceil \frac{\ell_B - \ell_P}{\ell_S} \rceil \), but clearly not true. Hence, the Rubber-stamping shareholder \( i \) cannot improve decision quality after vote recommendation for.

The Rubber-stamping shareholder \( i \) is pivotal after the against recommendation if the others’ votes are split. As \( \lceil \frac{\ell_B - \ell_P}{\ell_S} \rceil - 1 \) others play strategy Rubber-stamping, there are \( \lceil \frac{\ell_B - \ell_P}{\ell_S} \rceil - 1 \) unconditional yes-votes after recommendation against. The remaining \( N - \lceil \frac{\ell_B - \ell_P}{\ell_S} \rceil \) shareholders play strategy UNIS or strategy CAIS and are thus informed. Hence, \( i \) is pivotal after recommendation against if and only if \( \delta = \lceil \frac{\ell_B - \ell_P}{\ell_S} \rceil - 1 \). With Rubber-stamping, \( i \) would vote yes. To change the decision with a deviation she must deviate to voting no. This deviation would be most likely to increase decision quality if she had received signal \( a \). Decision quality would improve in that case if \( \ell_B < \ell_P + (\lceil \frac{\ell_B - \ell_P}{\ell_S} \rceil - 1)\ell_S + 1\ell_S \), which is equivalent to \( \frac{\ell_B - \ell_P}{\ell_S} < \lceil \frac{\ell_B - \ell_P}{\ell_S} \rceil \) and clearly not true, as \( |\ell_B - \ell_P| = \ell_B - \ell_P \) for \( q_B \geq q_P \) (which holds by assumption in Case i. of SOM Lemma 2.3). Therefore, a shareholder who Rubber-stamps cannot beneficially deviate.

The shareholders who play strategy UNIS bear the costs \( c \). The following information-acquisition strategies have lower costs (than NotSubscribe-Invest): NotSubscribe-NotInvest, Subscribe-NotInvest, Subscribe-InvestIFF for, and Subscribe-InvestIFF against, by the assumption that \( f \) is sufficiently lower than \( c \). Suppose one shareholder deviates to a strategy that involves one of these information-acquisition strategies. If this shareholder uses NotSubscribe-NotInvest, Subscribe-NotInvest, or Subscribe-InvestIFF against, then the number of informed shareholders after for is smaller than \( \bar{n}_1 \), namely \( \bar{n}_1 - 1 \), and hence C1 is violated. If this shareholders uses Subscribe-InvestIFF for, then the number of informed shareholders after recommendation against is smaller than \( \bar{n}_2 \), namely \( \bar{n}_2 - 1 \), and hence C2 is violated. Thus, any cost-saving deviation of a shareholder who plays strategy UNIS decreases decision quality. To improve decision quality, while at most \( \bar{n}_1 \) or \( \bar{n}_2 \) shareholders are informed after recommendation for or against, respectively, is not possible because decision quality is already maximal (by satisfying C1 and C2). To improve decision quality by deviating from strategy UNIS and increasing the number of informed shareholders after for or against is also not possible, as a shareholder who plays strategy UNIS is already informed in both cases. Thus, there is no beneficial deviation for a shareholder who plays strategy UNIS.

The shareholders who play strategy CAIS and hence use Subscribe-InvestIFF against could save costs by deviating and using the information-acquisition strategies NotSubscribe-NotInvest or Subscribe-NotInvest. Indeed, both would save costs by the assumption that \( f \) is sufficiently smaller than \( c \). Deviating to the other information-acquisition strategies would increase costs by the same assumption and by the fact that the recommendation for is more likely than against: \( q_Bq_P > q_B(1 - q_P) + (1 - q_B)q_P \). If a shareholder deviates to a strategy involving NotSubscribe-NotInvest or Subscribe-NotInvest, this reduces decision quality because the number of signals after recommendation against is
After the recommendation for, she votes yes according to CAIS and hence has to vote no and be pivotal to affect the decision with her deviation. After recommendation for, \([\ell_B + \ell_P] - 1\) shareholders vote yes without being informed when excluding \(i\) (summing up those who play strategy Rubber-stamping and those who play strategy CAIS, minus one player). Pivotality after recommendation for requires that the votes of the \(N - 1\) others are split, which occurs if and only if the signal difference of the \(n_1\) informed shareholders, \(\delta_{-i}\), satisfies \(\delta_{-i} = [\ell_B - \ell_P] - 1\) because this is the number of shareholders who vote yes without being informed. Conditioning on pivotality, voting no would be most likely to increase decision quality if the own signal was \(a\). Decision quality would improve in that case iff \(\ell_B + \ell_P < ([\ell_B + \ell_P] - 1)\ell_S + 1\ell_S\), which is equivalent to \(\ell_B + \ell_P < [\ell_B + \ell_P]\) and clearly not true.

After the against recommendation, a shareholder \(i\) who plays strategy CAIS votes according to the own signal, as do all others who play strategy CAIS or strategy UNIS. The \([\ell_B - \ell_P]\) Rubber-stamping shareholders vote yes. In order to affect the decision with her deviation, shareholder \(i\) has to vote yes after signal \(a\) or no after signal \(b\) and be pivotal. Pivotality after recommendation against requires that the votes of the \(N - 1\) others are split, which occurs if and only if \(\delta_{-i} = [\ell_B - \ell_P]\), where \(\delta_{-i}\) is again the signal difference against the proposal among the informed shareholders, excluding \(i\). Conditional on pivotality after recommendation against, deviating to voting yes after signal \(a\) would improve decision quality iff \(\ell_B > \ell_P + [\ell_B - \ell_P]\ell_S + 1\ell_S\), which is equivalent to \(\ell_B - \ell_P > [\ell_B - \ell_P] + 1\), which is clearly not true. Deviating to no after signal \(b\) would improve decision quality in that case iff \(\ell_B + 1\ell_S < \ell_P + [\ell_B - \ell_P]\ell_S\), which is equivalent to \(\ell_B - \ell_P + 1 < [\ell_B - \ell_P]\), which is clearly not true either. Thus, there is no beneficial deviation for a shareholder who plays strategy CAIS.

Taken together, above arguments imply that the proposed strategy profile is an equilibrium as no shareholder can profitably deviate. This equilibrium maximizes decision quality under the constraints of SOM Lemma 2.2 as it satisfies the Conditions C1 and C2 for these constraints. By SOM Lemma 2.2 there cannot be an equilibrium with looser constraints. Hence, among all equilibria it attains maximal decision quality. A Pareto-superior equilibrium would require lower costs for some shareholders without increasing the costs of any other shareholder. The total costs after vote recommendation for are \(\bar{n}_1c + ([\ell_B + \ell_P] - [\ell_B - \ell_P])f\); and after vote recommendation against, total costs are \(\bar{n}_2c + ([\ell_B + \ell_P] - [\ell_B - \ell_P])f\). Reducing the number of informed shareholders in either case to save costs \(c\) would lead to a violation of the Conditions C1 or C2. Reducing the number of shareholders who subscribe to the PA (by an integer \(t \in \{0, 1, \ldots, ([\ell_B + \ell_P] - [\ell_B - \ell_P])\}) to decrease costs related to fee \(f\) would either lead to a decrease in the number of informed shareholders or would require a “compensation” by having \(t\) more shareholders who unconditionally invest in an own signal (Subscribe-NotInvest). The former violates the Conditions C1 or C2, the latter is even more costly.
as $tc > tf$ holds by assumption. Thus, the proposed equilibrium is Pareto-efficient.

Finally, decision quality for this equilibrium can be constructed by using the probability that under the equilibrium strategy profile the decision is correct, considering the four combinations of whether the board’s signal is correct or wrong and whether the PA’s signal is correct or wrong, together with the probability $\pi(l, k)$ that among $l$ realizations with precision $q_S$ at least $k$ are correct: 

$$
\Pi^* = q_Bq_P\pi(N - [\frac{\ell_B + \ell_P}{\ell_S}], \frac{N+1}{2} - [\frac{\ell_B + \ell_P}{\ell_S}]) +
(1 - q_B)(1 - q_P)\pi(N - [\frac{\ell_B}{2}], \frac{N+1}{2} - [\frac{\ell_B}{2}]) +
q_B(1 - q_P)\pi(N - [\frac{\ell_B}{2}], \frac{N+1}{2} - [\frac{\ell_B}{2}]) +
(1 - q_B)q_P\pi(N - [\frac{\ell_B - \ell_P}{\ell_S}], \frac{N+1}{2}).
$$

ii. For $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B < \ell_P$, part i. of SOM Lemma 2.2 applies. Hence, $\bar{n}_1 = N - [\frac{\ell_B + \ell_P}{\ell_S}]$ and $\bar{n}_2 = N - [\frac{\ell_B - \ell_P}{\ell_S}]$. With the proposed strategy profile, we have $N - [\frac{\ell_B + \ell_P}{\ell_S}]$ shareholders who play strategy UNIS, which makes $\bar{n}_1$ informed shareholders after the for recommendation. After the against recommendation, we have additional $[\frac{\ell_B + \ell_P}{\ell_S}] - [\frac{\ell_B - \ell_P}{\ell_S}]$ informed shareholders who play strategy CAIS. This makes $N - [\frac{\ell_B - \ell_P}{\ell_S}] = \bar{n}_2$ informed shareholders after the against recommendation. The remaining $[\frac{\ell_B - \ell_P}{\ell_S}]$ shareholders play Follow, i.e., they are uninformed and always vote according to the vote recommendation.

Suppose the vote recommendation is for. Then, $[\frac{\ell_B + \ell_P}{\ell_S}]$ shareholders vote yes (those playing Follow plus those playing strategy CAIS), while $N - [\frac{\ell_B + \ell_P}{\ell_S}] = \bar{n}_1$ shareholders invest into a signal of quality $q_S$ and vote according to it. Thus, the proposal is accepted if and only if $\delta < [\frac{\ell_B + \ell_P}{\ell_S}]$, where $\delta$ is the signal difference against the board. Of the integers $[\frac{\ell_B + \ell_P}{\ell_S}]$ and $\delta$, exactly one is even and one is odd (see above). If $\delta < [\frac{\ell_B + \ell_P}{\ell_S}]$, then $\delta < [\frac{\ell_B - \ell_P}{\ell_S}]$ and the proposal is accepted, as required by C1. If $\delta > [\frac{\ell_B + \ell_P}{\ell_S}]$, then $\delta > \frac{\ell_B + \ell_P}{\ell_S} \ge [\frac{\ell_B - \ell_P}{\ell_S}]$ and the proposal is rejected, as required by C1. Hence, Condition C1 is satisfied.

After the against recommendation, the $[\frac{\ell_B - \ell_P}{\ell_S}]$ shareholders who play Follow vote no and all other shareholders (who play strategy CAIS or strategy UNIS) invest into an own signal and vote according to it. Thus, the proposal is accepted if and only if $\tilde{\delta} > [\frac{\ell_B - \ell_P}{\ell_S}]$, where $[\tilde{\delta}] = -\delta$ is the number of $b$-signals minus the number of $a$-signals (that is the vote difference in favor of the board). Of the integers $[\frac{\ell_B - \ell_P}{\ell_S}]$ and $\tilde{\delta}$ exactly one is even and one is odd. ($\tilde{\delta}$ is even if $\bar{n}_2$ is even, while $\bar{n}_2 = N - [\frac{\ell_B - \ell_P}{\ell_S}]$, whereas $N$ is always odd.) Thus, for any realization of signals we have $|\tilde{\delta} - [\frac{\ell_B - \ell_P}{\ell_S}]| \ge 1$. The condition $\tilde{\delta} > [\frac{\ell_B - \ell_P}{\ell_S}]$ is hence equivalent to $\delta > [\frac{\ell_B - \ell_P}{\ell_S}]$ (since $\ell_B < \ell_P$) to $-\delta > \frac{\ell_B - \ell_P}{\ell_S}$ and finally to $\delta < \frac{\ell_B - \ell_P}{\ell_S}$, as required by C2.

Hence, the proposed strategy profile satisfies Conditions C1 and C2. Potentially beneficial deviations must either reduce costs without affecting decision quality; or increase decision quality.

Consider a shareholder $i$ who plays Follow. The only possibility to save costs is to deviate to information-acquisition strategy NotSubscribe-NotInvest. Hence, only the
strategies Rubber-stamping and Protest save costs. Suppose \( i \) deviates to Rubber-stamping. Consider the realization of signals such that the vote recommendation is against and \( \tilde{\delta} = \frac{\ell_{B-\ell_P}}{\ell_S} - 1 \). By the assumption \( \frac{\ell_{B+\ell_P}}{\ell_S} < \frac{N+1}{2} \), we have \( \frac{\ell_{B-\ell_P}}{\ell_S} \leq \frac{\ell_{B+\ell_P}}{\ell_S} < \frac{N+1}{2} \) and \( \bar{n}_2 = N - \left( \frac{\ell_{B-\ell_P}}{\ell_S} \right) + 1 \) such that this realization occurs with positive probability. (Indeed, after recommendation against \( \tilde{\delta} \) attains all of the values in \( \left\{ -\left( N - \frac{\ell_{B-\ell_P}}{\ell_S} \right), \ldots, -\left( N - \frac{\ell_{B-\ell_P}}{\ell_S} \right) + 2, \ldots, N - \frac{\ell_{B-\ell_P}}{\ell_S} \right\} \) with positive probability.) Then excluding shareholder \( i \) we have \( \frac{N-1}{2} \) yes-votes and the same number of no-votes. Shareholder \( i \) is pivotal and makes the proposal accepted when Rubber-stamping. This violates Condition C2 because the proposal is accepted although \( \delta > \frac{\ell_{B-\ell_P}}{\ell_S} \). Indeed, \( \delta = \frac{\ell_{B-\ell_P}}{\ell_S} - 1 \) is equivalent to \( -\delta = \frac{\ell_{B-\ell_P}}{\ell_S} - 1 \) (because \( \delta = -\delta \) and \( \ell_B < \ell_P \)) and further equivalent to \( \delta = \frac{\ell_{B-\ell_P}}{\ell_S} + 1 \).

Now, suppose that \( i \) deviates to strategy Protest. Consider the realization of signals such that the vote recommendation is for or \( \tilde{\delta} = \frac{\ell_{B+\ell_P}}{\ell_S} - 1 \). As \( n_1 \geq \frac{N+1}{2} \geq \frac{\ell_{B+\ell_P}}{\ell_S} - 1 \), this realization occurs with positive probability. (Indeed, after recommendation for, \( \delta \) attains all of the values in \( \left\{ -(N - \frac{\ell_{B+\ell_P}}{\ell_S}), \ldots, -(N - \frac{\ell_{B+\ell_P}}{\ell_S}) + 2, \ldots, N - \frac{\ell_{B+\ell_P}}{\ell_S} \right\} \) with positive probability.) Then, excluding shareholder \( i \) we have \( \frac{N-1}{2} \) yes-votes and the same number of no-votes. Thus, shareholder \( i \) is pivotal and makes the proposal rejected when playing Protest. This violates Condition C1 because the proposal is rejected although \( \delta = \frac{\ell_{B-\ell_P}}{\ell_S} - 1 < \frac{\ell_{B+\ell_P}}{\ell_S} \).

Now, suppose that \( i \) deviates from Follow in order to increase decision quality. To affect the decision, \( i \) must either vote no after recommendation for or vote yes after recommendation against, or both. After for, the \( \left\lfloor \frac{\ell_{B-\ell_P}}{\ell_S} \right\rfloor - 1 \) other shareholders who Follow vote yes, as do the CAIS players. In sum, \( \left\lfloor \frac{\ell_{B+\ell_P}}{\ell_S} \right\rfloor - 1 \) shareholders vote yes when excluding \( i \), while the \( n_1 \) other shareholders play strategy UNIS and vote according to their signal. Hence, \( i \) is pivotal after recommendation for iff \( \delta = \frac{\ell_{B+\ell_P}}{\ell_S} - 1 \). Voting no after recommendation for would be most likely to increase decision quality if \( i \) had received signal \( a \). Decision quality would improve in that case iff \( \ell_B + \ell_P < \left( \frac{\ell_{B+\ell_P}}{\ell_S} - 1 \right) \ell_S + 1 \ell_S \), which is equivalent to \( \frac{\ell_{B+\ell_P}}{\ell_S} < \left\lfloor \frac{\ell_{B-\ell_P}}{\ell_S} \right\rfloor \) and clearly not true.

After recommendation against, the \( \left\lfloor \frac{\ell_{B-\ell_P}}{\ell_S} \right\rfloor - 1 \) shareholders who Follow vote no, while all \( \bar{n}_2 \) other shareholders (who play strategy CAIS or strategy UNIS) vote according to their signal. Hence, \( i \) is pivotal after recommendation against iff \( \tilde{\delta} = \left\lfloor \frac{\ell_{B-\ell_P}}{\ell_S} \right\rfloor - 1 \). Voting yes after recommendation against would be most likely to increase decision quality if \( i \) had received signal \( b \). Decision quality would improve in that case iff \( \ell_B + \left( \left\lfloor \frac{\ell_{B-\ell_P}}{\ell_S} \right\rfloor - 1 \right) \ell_S + 1 \ell_S > \ell_P \), which is equivalent to \( \left\lfloor \frac{\ell_{B-\ell_P}}{\ell_S} \right\rfloor \ell_S > \ell_P - \ell_B \) and with \( \ell_B < \ell_P \) also equivalent to \( \left\lfloor \frac{\ell_{B-\ell_P}}{\ell_S} \right\rfloor > \frac{\ell_P-\ell_B}{\ell_S} \), which is clearly not true.\(^{62}\) Therefore, a shareholder who plays Follow cannot beneficially deviate.

The shareholders who play strategy UNIS bear the costs \( c \). Any cost-saving deviation from strategy UNIS affects the number of informed shareholders either after recommendation for or after recommendation against or both. Hence, either Condition C1 or Condition C2 or both are violated in case of such a deviation. Therefore, any cost-saving deviation of a

\(^{62}\) Notice that we don’t apply the \( [\ldots] \)-operator to negative numbers.
shareholder from strategy UNIS decreases decision quality. To improve decision quality, while at most \(\bar{n}_1\) or \(\bar{n}_2\) shareholders are informed after recommendation \(for\) or \(against\), respectively, is not possible because decision quality is already maximal (by satisfying C1 and C2). To improve decision quality by increasing the number of informed shareholders after recommendation \(for\) or \(against\) is also not possible for a shareholder who plays strategy UNIS as she is already informed in both cases. Thus, there is no beneficial deviation for a shareholder who plays strategy UNIS.

The shareholders who play strategy CAIS and hence use Subscribe-InvestIFF\(against\) could save costs by the information-acquisition strategies NotSubscribe-NotInvest or Subscribe-NotInvest. If a shareholder deviates to a strategy involving one of these information-acquisition strategies (NotSubscribe-NotInvest or Subscribe-NotInvest), this reduces decision quality because the number of signals after recommendation \(against\) is smaller than \(\bar{n}_2\) and thus Condition C2 is violated. Hence, it remains to check whether a shareholder \(i\) who plays strategy CAIS can improve decision quality with some deviation. After recommendation \(for\), she votes \(yes\) according to CAIS. Moreover, the other shareholders who play strategy CAIS and those who play Follow vote \(yes\), which makes \(\frac{\ell_B+\ell_P}{\ell_S}\) unconditional \(yes\)-votes when excluding \(i\). The other \(\bar{n}_1\) shareholders play strategy UNIS and follow their signal. Shareholder \(i\) has to vote \(no\) and be pivotal to affect the decision after recommendation \(for\) with her deviation. Pivotality after recommendation \(for\) requires that the votes of the \(N-1\) others are split, which occurs if and only if the signal difference against the proposal of the (other) informed shareholders, \(\delta_{-i}\), satisfies \(\delta_{-i} = \lfloor \frac{\ell_B}{\ell_S} \rfloor - 1\) because this is the number of (other) shareholders who vote \(yes\) without being informed (they play Follow or strategy CAIS), excluding \(i\). Voting \(no\) would be most likely to increase decision quality when the own signal was \(a\). Decision quality would improve in that case iff \(\ell_B + \ell_P < (\lfloor \frac{\ell_B}{\ell_S} \rfloor - 1)\ell_S + 1\ell_S\), which is equivalent to \(\frac{\ell_B+\ell_P}{\ell_S} < \lfloor \frac{\ell_B}{\ell_S} \rfloor\) and clearly not true.

After the \(against\) recommendation, \(\lfloor \frac{\ell_B}{\ell_S} \rfloor\) shareholders who play Follow vote \(no\). A shareholder \(i\) who plays strategy CAIS votes according to her signal as do all others who play strategy CAIS or UNIS. In order to affect the decision with her deviation, shareholder \(i\) has has to vote \(yes\) after signal \(a\) or \(no\) after signal \(b\) and be pivotal. Pivotality after recommendation \(against\) requires that the votes of the \(N-1\) others are split, which occurs if and only if \(\delta_{-i} = \lfloor \frac{\ell_B}{\ell_S} \rfloor\), where \(\delta_{-i}\) is again the number of \(b\)-signals minus the number of \(a\)-signals, i.e., the signal difference in favor of the proposal, among the informed shareholders, excluding \(i\). Deviating to voting \(yes\) after signal \(a\) would improve decision quality in that case iff \(\ell_B + \frac{\ell_B}{\ell_S}\ell_S > \ell_P + 1\ell_S\), which is equivalent to \(\lfloor \frac{\ell_B}{\ell_S} \rfloor \ell_S > \ell_P - \ell_B + 1\ell_S\) and by \(\ell_B < \ell_P\) to \(\lfloor \frac{\ell_B}{\ell_S} \rfloor > \frac{\ell_P-\ell_B}{\ell_S} + 1\), which is clearly not true. Deviating to voting \(no\) after signal \(b\) would improve decision quality in case of pivotality iff \(\ell_B + \frac{\ell_B}{\ell_S}\ell_S + 1\ell_S < \ell_P\), which is equivalent to \(\lfloor \frac{\ell_B}{\ell_S} \rfloor \ell_S + 1\ell_S < \ell_P - \ell_B\) and by \(\ell_B < \ell_P\) to \(\lfloor \frac{\ell_B}{\ell_S} \rfloor + 1 < \frac{\ell_P-\ell_B}{\ell_S}\), which is clearly not true either.

Thus, there is no beneficial deviation for a shareholder who plays strategy CAIS either. Taken together, the above arguments imply that the proposed strategy profile is an
equilibrium, as there is no shareholder who can beneficially deviate. Moreover, it maximizes decision quality among all potential equilibria, as it satisfies the Conditions C1 and C2 for the constraints given by SOM Lemma 2.2. There cannot be a Pareto-superior equilibrium with higher decision quality. It remains to verify that there is no Pareto-superior equilibrium with the same decision quality, but lower costs. Any reduction of signal costs $c$ would lead to a violation of either Condition C1 or C2 or both. Any reduction of costs related to the fee $f$ would either also induce such a violation or would have to be compensated by shareholders who unconditionally invest $c$ into an own signal, which is more costly by the assumption that $f$ is sufficiently smaller than $c$. Hence, the equilibrium is Pareto-efficient.

iii. For $\frac{\ell_B - \ell_P}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S}$ and $\ell_B \geq \ell_P$, part iii. of SOM Lemma 2.2 applies. Hence, $\bar{n}_1 = 0$ and $\bar{n}_2 = N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$. With the proposed strategy profile, we have $N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ shareholders who play strategy CAIS and $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ who Rubber-stamp, which makes $\bar{n}_1 = 0$ informed shareholders after the for recommendation. After the against recommendation, we have $\bar{n}_2 = N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ informed shareholders who play strategy CAIS.

As $\bar{n}_1 = 0$, we have $\delta = 0$ after recommendation for. Since $\delta = 0 < \frac{\ell_B + \ell_P}{\ell_S}$, Condition C1 requires that the proposal is accepted. With the proposed strategy profile all $N$ shareholders vote yes after recommendation for such that the proposal is always accepted. Hence, Condition C1 is satisfied.

After the against recommendation, the $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ shareholders who play strategy Rubber-stamping vote yes, and all other shareholders (who play strategy CAIS) invest into an own signal and vote according to it. Thus, the proposal is accepted if and only if $\delta < \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$, where $\delta$ is again the number of $a$-signals minus the number of $b$-signals. Of the integers $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ and $\delta$ exactly one is even and one is odd. Thus, for any realization of signals we have $|\delta - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor| \geq 1$. The condition $\delta < \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ is hence equivalent to $\delta < \frac{\ell_B - \ell_P}{\ell_S}$ and (since $\ell_B \geq \ell_P$) to $\delta < \frac{\ell_B - \ell_P}{\ell_S}$, as required by C2.

Hence, the proposed strategy profile satisfies Conditions C1 and C2 and hence maximizes decision quality under the constraints that impose upper bounds $\bar{n}_1$ and $\bar{n}_2$. Potentially beneficial deviations must either reduce costs without affecting decision quality, or increase decision quality.

We describe the proposed strategy profile by $\hat{\sigma}_{\mu,\nu}$, where $\mu = \bar{n}_2 = N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ shareholders play strategy CAIS and the remaining $\nu = \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ shareholders play strategy Rubber-stamping. We have to show that no player has a deviation incentive. We begin with a shareholder who plays strategy Rubber-stamping, then turn to a shareholder who plays strategy CAIS.

A shareholder $j$ who plays strategy Rubber-stamping does not acquire any information and votes yes. In the strategy profile $\hat{\sigma}_{\mu,\nu}$, a Rubber-stamping player is pivotal iff the

\[63\text{Hence, } \delta = \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor \text{ is not possible.}\]
PA has recommended against and among the shareholders who play strategy CAIS, \( \nu - 1 \) more have received signal \( a \) than have received \( b \).

In order to improve decision quality by a deviation of shareholder \( j \), this deviation must change the voting outcome, i.e., \( j \) must vote no in some instance of pivotality. The most attractive starting point for voting no occurs when the PA recommends against and the own signal is \( a \). Suppose that shareholder \( j \) would then vote no. This is a strict improvement of decision quality iff \( (\nu - 1)\ell_s + \ell_p + \ell_S > \ell_B \), or equivalently iff \( \nu > \frac{\ell_B - \ell_p}{\ell_S} \). However, setting \( \nu = \lceil \frac{\ell_B - \ell_p}{\ell_S} \rceil \) precludes this. Hence, a Rubber-stamping shareholder cannot improve decision quality by deviating when \( \mu = \bar{n}_2 \). Considering that any deviation on the information-acquisition stages moreover means that more costs are incurred, there is no beneficial deviation for any Rubber-stamping shareholder.

Let us now turn to shareholders who play strategy CAIS. First, we show that there is no deviation that improves decision quality. We then proceed by showing that all deviations with identical decision quality come at the same or higher costs. Together, these two assertions then prove that no individual deviation can improve utility.

Concerning decision quality, it is obvious that improvements are impossible when the PA’s recommendation is for, as in this case all shareholders vote yes and no shareholder is pivotal, given that \( \lceil \frac{\ell_B - \ell_p}{\ell_S} \rceil < \frac{N - 1}{2} \) (and hence, the number of yes-votes of CAIS players is \( N - \lceil \frac{\ell_B - \ell_p}{\ell_S} \rceil > \frac{N + 1}{2} + 1 \)). In the special case of \( \lceil \frac{\ell_B - \ell_p}{\ell_S} \rceil = \frac{N - 1}{2} \), a CAIS player \( i \) is pivotal after recommendation for. A deviation to voting no would be most likely to increase decision quality if the own signal was \( a \). It would improve decision quality in that case iff \( \ell_B + \ell_p < 1\ell_S \), which is precluded by the assumption that \( \lceil \frac{\ell_B - \ell_p}{\ell_S} \rceil \geq \frac{N + 1}{2} > 1 \). Thus, a deviation may only improve decision quality by changing the outcome when the PA recommends against and the shareholder is pivotal. Pivotality implies that among the \( \mu - 1 \) other shareholders who play strategy CAIS, the difference between the numbers of \( a \) and \( b \)-signals must equal \( \nu \), the number of Rubber-stamping shareholders. Those signals are thus split into \( \frac{N - 1}{2} \) \( a \)-signals and \( \frac{N - 1}{2} - \nu \) \( b \)-signals. Shareholder \( i \) may improve decision quality by always voting yes in these instances if and only if \( \ell_B + (\frac{N - 1}{2} - \nu) \cdot \ell_S > \ell_p + \frac{N - 1}{2} \ell_S + \ell_S \), which is equivalent to \( \ell_B - \ell_p > (\nu + 1)\ell_S \) or \( \frac{\ell_B - \ell_p}{\ell_S} > \nu + 1 \), and thus to \( \nu < \frac{\ell_B - \ell_p}{\ell_S} - 1 \). This, however, is precluded by \( \nu = \lceil \frac{\ell_B - \ell_p}{\ell_S} \rceil \).

Similarly, shareholder \( i \) may improve decision quality by always voting no in these instances if and only if \( \ell_B + (\frac{N - 1}{2} - \nu) \cdot \ell_S + \ell_S < \ell_p + \frac{N - 1}{2} \ell_S \), which turns out to be equivalent to \( \nu > \frac{\ell_B - \ell_p}{\ell_S} + 1 \), which is again at odds with \( \nu = \lceil \frac{\ell_B - \ell_p}{\ell_S} \rceil \). Thus, it is impossible to improve decision quality by a deviating strategy.

The only possibility remaining for a CAIS player in order to improve utility is thus to look for strategies that attain the same decision quality as \( \hat{\sigma}^{\mu,\nu} \), but at lower costs. As the costs associated with \( \hat{\sigma}^{\mu,\nu} \) are the fee \( f \) as well as costs \( c \) in case of the PA recommending against, there are two possibilities for reducing costs: the first one would be to get rid of conditional costs \( c \), which, however, would result in reduced decision quality, as such a deviation violates Condition C2. The second alternative is to get rid of the fee \( f \), by always voting according to the own signal. While this preserves the maximal decision quality and saves fee \( f \), it comes at additional costs \( c \) when the PA recommends for.
(Thus, in expectation, costs decrease by $f$, but increase by $c$ times the probability of a for recommendation (which is $q_B q_P + (1 - q_B)(1 - q_P)$.) As we have assumed that $f$ is sufficiently smaller than $c$, this is an overall increase of costs.

Taken together, we have shown that there is no utility improving deviation for the CAIS players, and neither for the Rubber-stamping players. Hence, the proposed strategy profile is an equilibrium. Moreover, it maximizes decision quality among all potential equilibria, as it satisfies the Conditions C1 and C2 for the constraints given by SOM Lemma 2.2. There cannot be a Pareto-superior equilibrium with higher decision quality. It remains to verify that there is no Pareto-superior equilibrium with the same decision quality, but lower costs. Any reduction of signal costs $c$ would lead to a violation of Condition C2. Any reduction of costs related to the fee $f$ would either also induce such a violation or had to be compensated by shareholders who unconditionally invest in a signal $c$, which is more costly by the assumption that $f$ is sufficiently smaller than $c$. Hence, the strategy profile is Pareto-efficient.

Finally, decision quality follows again from the probability that under the equilibrium strategy profile the decision is correct, considering the four combinations of whether the board’s signal is correct or wrong and whether the PA’s signal is correct or wrong, together with the probability $\pi(l, k)$ that among $l$ realizations with precision $q_S$ at least $k$ are correct: $\Pi(\sigma) = q_B q_P + q_B (1 - q_P)\pi(z_2, z_2 - \frac{N - 1}{2}) + (1 - q_B)q_P\pi(z_2, \frac{N + 1}{2}) > q_B$.

iv. For $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N + 1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S}$ and $\ell_B < \ell_P$, part ii. of SOM Lemma 2.2 applies. Hence, $\bar{n}_1 = 0$ and $\bar{n}_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$. With the proposed strategy profile, we have $N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders who play strategy CAIS and $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ who play strategy Protest, which makes $\bar{n}_1 = 0$ informed shareholders after the for recommendation. After the against recommendation, we have $\bar{n}_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ informed shareholders who play strategy CAIS.

As $\bar{n}_1 = 0$, we have $\delta = 0$ after recommendation for. Since $\delta = 0 < \ell_B + \ell_P$, Condition C1 requires that the proposal is accepted. With the proposed strategy profile, there are $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ no-votes by the Protest players versus $N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ yes-votes by the CAIS players after recommendation for. By the assumption $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N + 1}{2}$, we have $\bar{n}_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor \geq \frac{N + 1}{2} > \frac{|\ell_B - \ell_P|}{\ell_S}$ such that the proposal is always accepted after recommendation for, as required by Condition C1.

After the against recommendation, the $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders who play strategy Protest vote unconditionally no and all other shareholders (who play strategy CAIS) invest into an own signal and vote according to it. Thus, the proposal is accepted if and only if $\tilde{\delta} > \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$, where $\tilde{\delta} = -\delta$ is again the number of $b$-signals minus the number of $a$-signals, i.e., the signal difference in favor of the proposal. Of the integers $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ and $\tilde{\delta}$, exactly one is even and one is odd. Thus, for any realization of signals we have $|\tilde{\delta} - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor| \geq 1$. The condition $\tilde{\delta} > \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ is hence equivalent to $\tilde{\delta} > \frac{|\ell_B - \ell_P|}{\ell_S}$ and (since $\ell_B < \ell_P$) to $-\delta > \frac{\ell_P - \ell_B}{\ell_S}$ and finally to $\delta < \frac{\ell_B - \ell_P}{\ell_S}$, as required by C2.

Hence, the proposed strategy profile satisfies Conditions C1 and C2. Potentially beneficial deviations must either reduce costs without affecting decision quality, or increase decision
quality.

Consider a shareholder $i$ who plays Protest. There is no possibility to save costs as all other information-acquisition strategies are more costly than NotSubscribe-NotInvest. Now, suppose $i$ deviates from strategy Protest in order to increase decision quality.

After vote recommendation for, a majority of $n_2 = N - \left\lfloor \frac{t_B - t_P}{\ell_S} \right\rfloor \geq \frac{N+1}{2}$ shareholders (who play strategy CAIS) vote yes. Hence, shareholder $i$ who plays Protest is not pivotal and hence cannot affect decision quality.

After recommendation against, the $\left\lceil \frac{t_B - t_P}{\ell_S} \right\rceil - 1$ other shareholders who play strategy Protest vote no, while all $\tilde{n}_2$ shareholders who play strategy CAIS vote according to their signal. Hence, $i$ is pivotal after recommendation against iff $\tilde{\delta} = \left\lceil \frac{t_B - t_P}{\ell_S} \right\rceil - 1$, where again $\tilde{\delta} = -\delta$. Under playing Protest, $i$ would vote no. To effectively deviate, $i$ has to be pivotal and vote yes. Voting yes after recommendation against would be most likely to increase decision quality if $i$ had received signal $b$. Decision quality would improve in that case iff $\ell_B + (\left\lfloor \frac{t_B - t_P}{\ell_S} \right\rfloor - 1)\ell_S + 1\ell_S > t_P$, which is equivalent to $\left\lfloor \frac{t_B - t_P}{\ell_S} \right\rfloor \ell_S > t_P - \ell_B$ and with $\ell_B < t_P$ also equivalent to $\left\lfloor \frac{t_P - t_B}{\ell_S} \right\rfloor > \frac{t_P - t_B}{\ell_S}$, which is clearly not true. Therefore, a shareholder who plays strategy Protest cannot beneficially deviate.

It remains to show that shareholders who play strategy CAIS cannot beneficially deviate. First, we address possibilities to improve by reducing costs at the same decision quality, then we turn to deviations intended to increase decision quality.

As the costs associated with the proposed strategy profile are the fee $f$ as well as costs $c$ in case of the PA recommending against, there are two possibilities for reducing costs: the first one would be to get rid of conditional costs $c$, which, however, would result in reduced decision quality, as it violates Condition C2. The second alternative is to get rid of the fee $f$, by always voting according to the individual signal. While this preserves the optimal decision quality and saves fee $f$, it comes with additional costs $c$ when the PA recommends for. As we have assumed that $f$ is sufficiently smaller than $c$, this is an overall increase of costs.

Consider a shareholder $i$ who plays strategy CAIS. After recommendation for, the other CAIS players vote yes while the Protest players vote no such that we have $N - \left\lfloor \frac{t_B - t_P}{\ell_S} \right\rfloor - 1$ yes-votes when excluding $i$, and $\left\lfloor \frac{t_B - t_P}{\ell_S} \right\rfloor$ no-votes. If $\left\lfloor \frac{t_B - t_P}{\ell_S} \right\rfloor < \frac{N-1}{2}$, then there is a majority voting yes and $i$ is not pivotal. In that case, $i$ cannot affect decision quality after recommendation for. Otherwise, we have $\left\lfloor \frac{t_B - t_P}{\ell_S} \right\rfloor = \frac{N-1}{2}$. Then $i$ is pivotal after recommendation for. Under strategy CAIS, $i$ votes yes and the proposal is accepted. To affect the outcome after recommendation for by deviating, $i$ has to vote no and be pivotal under this deviation. Such a deviation would be most likely to increase decision quality if $i$ had received signal $a$. This deviation would increase decision quality in that case iff $\ell_B + \ell_P < 1\ell_S$, which is equivalent to $\frac{\ell_B + \ell_P}{\ell_S} < 1$ and precluded by the assumption $\frac{\ell_B + \ell_P}{\ell_S} \geq \frac{N+1}{2} > 1$.

After recommendation against, shareholder $i$ who plays strategy CAIS votes according to the own signal, as do all others who play strategy CAIS, while the $\left\lfloor \frac{t_B - t_P}{\ell_S} \right\rfloor$ Protest players vote no. In order to affect the decision with her deviation, shareholder $i$ has to vote yes after signal $a$ or no after signal $b$ and be pivotal. Pivotality after recommendation
against requires that the votes of the $N - 1$ others are split, which occurs if and only if
\[ \tilde{\delta}_{-i} = \left\lceil \frac{[|l_B - \ell_P|]}{l_S} \right\rceil \]
where \( \tilde{\delta}_{-i} \) is again the number of \( b \)-signals minus the number of \( a \)-signals, i.e., the signal difference in favor of the proposal, among the informed shareholders, excluding \( i \). Deviating to voting yes after signal \( a \) would improve decision quality in that case iff \( \ell_B + \left\lceil \frac{|l_B - \ell_P|}{l_S} \right\rceil \ell_S > \ell_P + 1 \ell_S \), which is equivalent to \( \left\lceil \frac{|l_B - \ell_P|}{l_S} \right\rceil \ell_S > \ell_P - \ell_B + 1 \ell_S \) and by \( \ell_B < \ell_P \) to \( \left\lceil \frac{\ell_P - \ell_B}{l_S} \right\rceil + 1 \), which is clearly not true. Deviating to no after signal \( b \) would improve decision quality in case of pivotality iff \( \ell_B + \left\lceil \frac{|l_B - \ell_P|}{l_S} \right\rceil \ell_S + 1 \ell_S < \ell_P \), which is equivalent to \( \left\lceil \frac{|l_B - \ell_P|}{l_S} \right\rceil + 1 < \frac{\ell_P - \ell_B}{\ell_S} \), which is clearly not true either. Hence, a shareholder who plays strategy CAIS cannot beneficially deviate either.

Taken together, we thus have shown that there is no utility improving deviation strategy for the CAIS players, and neither for the Protest players. Hence, the proposed strategy profile is an equilibrium. Moreover, it maximizes decision quality among all potential equilibria, as it satisfies the Conditions C1 and C2 for the constraints given by SOM Lemma 2.2. There cannot be a Pareto-superior equilibrium with higher decision quality. It remains to verify that there is no Pareto-superior equilibrium with the same decision quality, but lower costs. Any reduction of signal costs \( c \) would lead to a violation of Condition C2. Any reduction of costs related to the fee \( f \) would either also induce such a violation or would have to be compensated by shareholders who unconditionally invest \( c \) into an own signal, which is more costly by the assumption that \( f \) is sufficiently smaller than \( c \). Hence, the equilibrium is Pareto-efficient.

Finally, decision quality follows again from the probability that under the equilibrium strategy profile the decision is correct, considering the four combinations of whether the board’s signal is correct or wrong and whether the PA’s signal is correct or wrong, together with the probability \( \pi(l,k) \) that that among \( l \) realizations with precision \( q_S \) at least \( k \) are correct: The decision quality in this profile is \( \Pi^* = q_B q_P + q_B (1 - q_P) \pi(N - \left\lceil \frac{|l_B - \ell_P|}{l_S} \right\rceil, N + \frac{1}{2}) + (1 - q_B)q_P \pi(N - \left\lceil \frac{|l_B - \ell_P|}{l_S} \right\rceil, N + \frac{1}{2} - \left\lceil \frac{|l_B - \ell_P|}{l_S} \right\rceil). \)

v. For \( \left\lceil \frac{|l_B - \ell_P|}{l_S} \right\rceil \geq \frac{N+1}{2} \) and \( \ell_B \geq \ell_P \), part iii. of SOM Lemma 2.2 applies (with \( \ell_B > \ell_P \), since zero cannot be larger than or equal to \( \frac{N+1}{2} \)). Hence, \( \tilde{n}_1 = \tilde{n}_2 = 0 \). By definition the signal difference is \( \delta = 0 \) when there are no informed shareholders. Hence, Condition C1 requires to accept the board’s proposal after recommendation for iff \( 0 = \delta < \frac{\ell_B + \ell_P}{l_S} \), which is always satisfied. Condition C2 requires to accept the proposal after recommendation against iff \( 0 = \delta < \frac{|l_B - \ell_P|}{\ell_S} \), which is also always satisfied, as we have \( \frac{|l_B - \ell_P|}{l_S} \geq \frac{N+1}{2} > 0 \). Hence, to maximize decision quality, the proposal must be accepted.

In the proposed strategy profiles, \( N' \in \{ \frac{N+1}{2}, \ldots, N \} \) shareholders play strategy Rubber-stamping and \( N - N' \) play strategy Protest. The \( N' \) Rubber-stamper vote yes, while the \( N - N' \) shareholders who play strategy Protest vote no. As \( N' \geq \frac{N+1}{2} > N - N' \), the proposal is accepted. Hence, any of these strategy profiles satisfies C1 and C2 and thus maximizes decision quality.

It remains to show that there are no profitable deviations. First, neither a Rubber-stamping shareholder nor a Protest player can save costs, as NotSubscribe-NotInvest has no costs. A Protest playing shareholder cannot improve decision quality as she
is not pivotal. Indeed, there are at least \( N' \geq \frac{N+1}{2} \) yes-votes. If \( N' \geq \frac{N+1}{2} + 1 \), then no Rubber-stamping shareholder \( i \) is pivotal either, as there are at least \( \frac{N+1}{2} \) yes-votes without \( i \). Otherwise, i.e., if \( N' = \frac{N+1}{2} \), a Rubber-stamping shareholder \( i \) is pivotal, as there are \( \frac{N-1}{2} \) yes-votes, excluding \( i \), and the same number of no-votes. To change the outcome by a deviation, shareholder \( i \) has to vote no. This deviation would be most likely to increase decision quality after the recommendation against if \( i \) had received signal \( a \). This deviation would improve decision quality in that case iff \( \ell_B < \ell_P + 1 \ell_S \), which is equivalent to \( \frac{\ell_B - \ell_P}{\ell_S} < 1 \), but precluded by \( \frac{\ell_B - \ell_P}{\ell_S} = \frac{|\ell_B - \ell_P|}{\ell_S} \geq \frac{N+1}{2} > 1 \) (for \( \ell_B \geq \ell_P \)).

Therefore, the proposed strategy profiles are equilibria. Moreover, they are not Pareto-dominated by any other equilibrium because they not only maximize decision quality under the constraints of SOM Lemma 2.2, but also lead to no costs.

Decision quality in such a strategy profile equals the probability that the board has received the correct signal, i.e., \( \Pi^* = q_B \).

vi. For \( \frac{|\ell_B - \ell_P|}{\ell_S} \geq \frac{N+1}{2} \) and \( \ell_B < \ell_P \), part iii. of SOM Lemma 2.2 applies. Hence, \( \bar{n}_1 = \bar{n}_2 = 0 \). By definition the signal difference is \( \delta = 0 \) when there are no informed shareholders. Hence, Condition C1 requires to accept the board’s proposal after recommendation for iff \( 0 = \delta < \frac{\ell_B + \ell_P}{\ell_S} \), which is always satisfied. Condition C2 requires to reject after recommendation against iff \( \delta < \frac{\ell_B - \ell_P}{\ell_S} \) which is also always satisfied, as we have \( \delta = 0 \) and \( \ell_B - \ell_P < 0 \). Hence, to maximize decision quality, the proposal must be accepted if and only if the vote recommendation is for.

In the proposed strategy profiles, \( \frac{N-1}{2} \) shareholders play strategy Rubber-stamping and \( \frac{N-1}{2} \) play strategy Protest, while one shareholder plays Follow. The votes of the Rubber-stamping shareholders and the votes of the Protest players cancel each other out, while the shareholder who plays Follow determines the outcome according to the vote recommendation. Hence, this strategy profile satisfies both C1 and C2 and thus maximizes decision quality.

It remains to show that there are no profitable deviations. First, neither a Rubber-stamping shareholder nor a Protest player can save costs, as NotSubscribe-NotInvest does not involve any costs. A Rubber-stamping player \( i \) cannot improve decision quality after recommendation against because she is not pivotal after it. (Indeed, after recommendation against, there are \( \frac{N-1}{2} + 1 \) no-votes by the Protest-voters and the Follow-voter.) To change the outcome, shareholder \( i \) has to vote no after recommendation for. This deviation would be most likely to increase decision quality if \( i \) had received signal \( a \). This deviation would improve decision quality in that case iff \( \ell_B + \ell_P < 1 \ell_S \), which is equivalent to \( \frac{\ell_B + \ell_P}{\ell_S} < 1 \), but precluded by \( \frac{\ell_B + \ell_P}{\ell_S} > \frac{|\ell_B - \ell_P|}{\ell_S} \geq \frac{N+1}{2} > 1 \). Hence, a Rubber-stamping shareholder cannot beneficially deviate.

A Protest playing shareholder \( i \) cannot improve decision quality after recommendation for as she is not pivotal. (Indeed, after recommendation for, there are at least \( \frac{N-1}{2} + 1 \) yes-votes and \( \frac{N-1}{2} - 1 \) no-votes when excluding \( i \).) To change the outcome, shareholder \( i \) has to vote yes after recommendation against. This deviation is most attractive if \( i \) had received signal \( b \). This deviation would improve decision quality in that case iff \( \ell_B + 1 \ell_S > \ell_P \),
which is equivalent to $1 > \frac{\ell_P - \ell_B}{\ell_s}$, but precluded by $\frac{\ell_P - \ell_B}{\ell_s} = \frac{|\ell_B - \ell_P|}{\ell_s} \geq \frac{N+1}{2} > 1$ (for $\ell_B < \ell_P$).

Now, consider the shareholder who plays Follow. She could save costs, which consist of the fee $f$, only by switching to information-acquisition strategy NotSubscribe-NotInvest. Without subscribing to the vote recommendation, $i$ cannot condition the voting behavior on it. Therefore, either $C1$ or $C2$ or both are violated when saving costs. Hence, decision quality suffers, and this type of deviation is not beneficial. Now, suppose that $i$ who plays Follow deviates in order to increase decision quality. To change the outcome, shareholder $i$ has to vote $yes$ after recommendation against and/or $no$ after recommendation for. A deviation including the former would be most likely to increase decision quality if $i$ had invested into an own signal and received signal $b$. This deviation would improve decision quality in that case iff $\ell_B + 1\ell_s > \ell_P$, which is equivalent to $1 > \frac{\ell_P - \ell_B}{\ell_s}$, but precluded as shown already above. A deviation including the latter behavior, i.e., voting $no$ after recommendation for, would be most likely to increase decision quality if $i$ had invested into an own signal and received signal $a$. This deviation would improve decision quality in that case iff $\ell_B + \ell_P < 1\ell_s$, which is equivalent to $\frac{\ell_P + \ell_B}{\ell_s} < 1$, but precluded as shown already above.

Therefore, the proposed strategy profile is an equilibrium. Moreover, by satisfying $C1$ and $C2$ for $\bar{n}_1 = \bar{n}_2 = 0$ it maximizes decision quality under the constraints given by SOM Lemma 2.2. Costs for all shareholders together are $f$. Any strategy profile with lower costs necessarily violates $C1$ or $C2$ and hence induces lower decision quality. Thus, the proposed strategy profile cannot be Pareto-dominated by any other equilibrium.

Decision quality in such a strategy profile equals the probability that the PA has received the correct signal, i.e., $\Pi^* = q_P$.

\[ \square \]

### 2.3 Research Incentives Increase with a Proxy Advisor

Analogously to the analysis of symmetric equilibria in Section 3, the negative result obtained without a PA can be mitigated when a PA is admitted. Again, the basic idea is that the PA’s recommendation is used as a condition to invest in own research like in information-acquisition strategy Subscribe-InvestIFF against, which constitutes CAIS. While this was true for all shareholders in Proposition 2 in a certain parameter range, we now find that in much larger areas of the parameter space some, but not all, shareholders use this strategy. Based on SOM Lemmata 2.1, 2.2 and 2.3, the following result summarizes the number of informed and conditionally informed shareholders in asymmetric equilibria with a PA.

**Proposition 2.2** (ASYM with PA). Let Assumption PAF hold. Let costs $c > 0$ be arbitrarily small and let fee $f$ be sufficiently smaller. Suppose there is a PA with information quality $q_P$ such that $\frac{|\ell_B - \ell_P|}{\ell_s} < \frac{N+1}{2}$. Then there exists an equilibrium in which the number of shareholders who invest or conditionally invest in own research is $z_2 \geq \frac{N+1}{2}$, with $z_2 := N - \left[ \frac{|\ell_B - \ell_P|}{\ell_s} \right]$. In particular, in the Pareto-efficient equilibria the following holds:
i. If $\frac{t_B + t_P}{t_s} < \frac{N+1}{2}$, then $N - \left\lfloor \frac{t_B - t_P}{t_s} \right\rfloor \approx \frac{2t_P}{t_s}$ shareholders conditionally invest in own research, playing strategy CAIS, and $N - \left\lfloor \frac{t_B + t_P}{t_s} \right\rfloor$ shareholders unconditionally invest in own research, playing strategy UNIS.

ii. If $\frac{t_B - t_P}{t_s} < \frac{N+1}{2} \leq \frac{t_B + t_P}{t_s}$, then $N - \left\lfloor \frac{t_B - t_P}{t_s} \right\rfloor$ shareholders conditionally invest in own research, playing strategy CAIS, and no shareholder unconditionally invests in own research.

iii. Otherwise, i.e., if $\frac{N+1}{2} \leq \frac{t_B - t_P}{t_s}$, no shareholder invests in own research.

**Proof.** To prove SOM Proposition 2.2, we use SOM Lemma 2.3 established in Section 2.2.

Suppose $\frac{t_B - t_P}{t_s} < \frac{N+1}{2}$. Then either one of the cases i.-iv. of SOM Lemma 2.3 applies. In each of these, there is an equilibrium where the number of CAIS players plus the number of UNIS players equals $N - \left\lfloor \frac{t_B - t_P}{t_s} \right\rfloor$. Indeed, in cases i. and ii. of SOM Lemma 2.3, we have $\left\lfloor \frac{t_B + t_P}{t_s} \right\rfloor - \left\lfloor \frac{t_B - t_P}{t_s} \right\rfloor + N - \left\lfloor \frac{t_B + t_P}{t_s} \right\rfloor = N - \left\lfloor \frac{t_B - t_P}{t_s} \right\rfloor$ shareholders who invest or conditionally invest in own research. In cases iii. and iv. of SOM Lemma 2.3, $N - \left\lfloor \frac{t_B - t_P}{t_s} \right\rfloor$ shareholders play strategy CAIS and hence conditionally invest.

Now, we address part i. For $\frac{t_B + t_P}{t_s} < \frac{N+1}{2}$, either case i. or ii. of SOM Lemma 2.3 applies. In both cases, the assertion follows directly from the lemma. In the Pareto-efficient equilibria, $\left\lfloor \frac{t_B + t_P}{t_s} \right\rfloor - \left\lfloor \frac{t_B - t_P}{t_s} \right\rfloor \approx \frac{2t_P}{t_s}$ shareholders conditionally invest in own research, playing strategy CAIS, and $N - \left\lfloor \frac{t_B + t_P}{t_s} \right\rfloor$ shareholders unconditionally invest in own research, playing UNIS.

Second, we address part ii. For $\frac{t_B - t_P}{t_s} < \frac{N+1}{2} \leq \frac{t_B + t_P}{t_s}$, either case iii. or iv. of SOM Lemma 2.3 applies. In both cases, the assertion follows directly from the lemma. ($N - \left\lfloor \frac{t_B - t_P}{t_s} \right\rfloor$ shareholders conditionally invest in own research, playing strategy CAIS in the Pareto-efficient equilibrium.)

Finally, we address part iii. For $\frac{N+1}{2} \leq \frac{t_B - t_P}{t_s}$, case v. of Lemma 2.3 applies. The assertion of part iii. follows directly from it. 

The proposition states that there exists an equilibrium in which more than half of all shareholders invest or conditionally invest in own research, given that the PA’s information quality is not too different from the board’s. More precisely, the condition is $\frac{t_B - t_P}{t_s} < \frac{N+1}{2}$, which means that the difference between the information quality of the PA and the information quality of the board must not exceed the aggregated information quality of about half of all shareholders together. (Observe that the larger the number of shareholders $N$, the less demanding this assumption is.) Under this condition, $z_2 = N - \left\lfloor \frac{t_B - t_P}{t_s} \right\rfloor$ shareholders invest or conditionally invest in research. Compared to the number of shareholders who invest in research without a PA (SOM Proposition 2.2), we have $z_2 \geq z_1$, or $N - \left\lfloor \frac{t_B - t_P}{t_s} \right\rfloor \geq N - \left\lfloor \frac{t_B}{t_s} \right\rfloor$, with a strict difference, e.g., if $q_P > q_s$. Hence, due to the presence of the PA, there are more shareholders who invest or conditionally invest under these conditions. The term $\frac{t_B - t_P}{t_s}$ measures the difference between the information qualities of the board and the PA relative to a single shareholder’s. The larger this difference, the lower the number $z_2$ of shareholders who invest or conditionally invest. It starts with $z_2 = N$ when the difference is close to zero, i.e., $\left\lfloor \frac{t_B - t_P}{t_s} \right\rfloor = 0$, and decreases down to $z_2 = \frac{N+1}{2}$ when $\left\lfloor \frac{t_B - t_P}{t_s} \right\rfloor = \frac{N-1}{2}$. Hence, the number of shareholders who invest or conditionally invest is maximized for $q_P \approx q_B$. 

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SOM Proposition 2.2 then further characterizes the Pareto-efficient equilibria distinguishing three cases. The lower panel of Figure 2.2 illustrates for each of the three cases, how many shareholders are investing in research, followed by how many are conditionally investing in research. The first two cases both satisfy assumption \( \left[ \ell_B - \ell_P \right] t_s < \frac{N+1}{2} \), i.e., that the PA’s and the board’s signal quality are not too different. The cases differ in that the joint decision quality of PA and board is assumed to be smaller than the decision quality of half of all shareholders in case i., i.e., \( \frac{\ell_B + \ell_P}{t_s} < \frac{N+1}{2} \), but not in case ii. In the lower panel of Figure 2.2, case i. corresponds to bottom left triangle and case ii. corresponds to the area between the two parallel lines. In case i., a majority of \( N - \left[ \frac{\ell_B - \ell_P}{t_s} \right] \) shareholders unconditionally invests in own research. In case ii., a larger majority of \( N - \left[ \frac{\ell_B - \ell_P}{t_s} \right] \) shareholders conditionally invests in own research (while in both cases in total \( z_2 = N - \left[ \frac{\ell_B - \ell_P}{t_s} \right] \) shareholders invest or conditionally invest).

Moreover, the upper panel of SOM Figure 2.2 illustrates how many shareholders are investing in own research in the Pareto-efficient equilibria for the benchmark setting in which the PA is not admitted, as established by SOM Proposition 2.1. We can observe the following effects when admitting a PA: in some areas (e.g., in the lower right), the number of shareholders who invest or conditionally invest in own research remains zero. In some other areas (e.g., in the upper right) this number increases from 0 to \( N - \left[ \frac{\ell_B - \ell_P}{t_s} \right] \) who conditionally invest in own research. In the most interesting regions (in the lower left and center), the number of shareholders changes from \( N - \left[ \frac{\ell_B - \ell_P}{t_s} \right] \) who invest without a PA, to \( N - \left[ \frac{\ell_B - \ell_P}{t_s} \right] \) of whom either all conditionally invest or of whom \( N - \left[ \frac{\ell_B - \ell_P}{t_s} \right] \) invest and the other \( \approx 2 \ell_P/s_B \) conditionally invest (cases i. and ii. of SOM Proposition 2.2). Finally, there is the area in the upper left, where the number of shareholders who invest reduces from \( N - \left[ \frac{\ell_B + \ell_P}{t_s} \right] \) to 0.

The change in research incentives can be seen even more specifically, when returning to SOM Figure 2.1. In the lower panel the three cases of SOM Proposition 2.2 are further split into six regions, according to whether \( \ell_B > \ell_P \), i.e., whether we are below or above the 45-degree line. For instance, in the bottom left triangle, defined by \( \frac{\ell_B + \ell_P}{t_s} < \frac{N+1}{2} \) and \( \ell_B \geq \ell_P \) (case i. of SOM Lemma 2.3), in the Pareto-efficient equilibrium \( \left[ \frac{\ell_B - \ell_P}{t_s} \right] \) play strategy Rubber-stamping, \( \left[ \frac{\ell_B + \ell_P}{t_s} \right] - \left[ \frac{\ell_B - \ell_P}{t_s} \right] \) play strategy CAIS, and \( N - \left[ \frac{\ell_B + \ell_P}{t_s} \right] \) play strategy UNIS. In comparison to the Pareto-efficient equilibrium without a PA, in which \( \left[ \frac{\ell_B}{t_s} \right] \) play strategy Rubber-stamping and \( N - \left[ \frac{\ell_B}{t_s} \right] \) play strategy UNIS, this means that due to the presence of the PA, we gain about \( 2 \ell_P/s_B \) who play strategy CAIS, about half of which would play strategy Rubber-stamping without a PA and half of which would play strategy UNIS without a PA. Above the 45-degree line there can be shareholders who play Subscribe-NotInvest and always follow the recommendation. We call this strategy Follow, as these shareholders buy the PA’s recommendation and follow it when voting. The only decreasing line incorporates the condition \( \frac{\ell_B + \ell_P}{t_s} < \frac{N+1}{2} \), which is necessary and sufficient for

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64 This triangle is not the small lower left triangle at \( x = 1 \) and \( y = 1 \) which in the case of symmetric equilibria admits UNIS and is excluded by Assumption BIB, but a much larger area of the parameter space, as it goes up to \( \frac{N+1}{2} \).

65 As Proposition 3 implies, the overall effect of the PA on decision quality is however still positive, even in this range.
Figure 2.2: Number of shareholders who invest or conditionally invest. The upper panel depicts the game without a PA based on SOM Proposition 2.1. The lower panel depicts the game with a PA based on SOM Proposition 2.2.
having shareholders who unconditionally invest, i.e., play strategy UNIS.

Overall, the conditions for the existence of an equilibrium with information acquisition by a majority of shareholders are relaxed (SOM Proposition 2.2), compared with those for a symmetric equilibrium in which all shareholders acquire information (Proposition 2). In fact, we move from the requirement that the normalized difference in expertise between board and PA equals at most one shareholder in the symmetric case to the corresponding requirement for the asymmetric case that this difference equals at most half of all shareholders, approximately. In sum, the novel type of equilibrium behavior that we find in this paper exists in a broad range of the parameter space.

2.4 Decision Quality Improves with a Proxy Advisor

For asymmetric equilibria, Proposition 3 in the main text establishes generally that the introduction of a PA weakly improves decision quality. More specifically, SOM Proposition 2.1 provides the Pareto-efficient equilibria in the benchmark setting when no PA is admitted and SOM Lemma 2.3 provides the Pareto efficient equilibria with a PA.

To illustrate the quantitative difference in decision quality, we consider a simple numerical example.

Example 2.1 (Asymmetric Equilibria). Let \( q_S = 0.6, q_B = 0.8, \) and \( q_P = 0.7 \).\(^{66}\) Then, \( \ell_B/\ell_S = 3.4 \) and \( \ell_P/\ell_S = 2.1 \), and the case distinction in SOM Proposition 2.1, which characterizes the benchmark scenario in which no PA is admitted, has a threshold at \( N = 5.8 \). Hence, for \( N \leq 5 \), part i. of SOM Proposition 2.1 applies, while for \( N \geq 6 \) part ii. applies. Thus, either all shareholders are uninformed or at most \( z_1 = N - \lfloor \ell_B/\ell_S \rfloor = N - 3 \) invest in own research when there is no PA.

Concerning the game with a PA, the condition \( \lfloor (\ell_B - \ell_P)/\ell_S \rfloor < \frac{N+1}{2} \) of SOM Proposition 2.2 is satisfied. Hence, we get existence of an equilibrium with \( z_2 = N - \lfloor |\ell_B - \ell_P|/\ell_S \rfloor = N - 1 \) shareholders who invest, or conditionally invest, in an own signal. More precisely, for \( N \leq 10 \), part i. of SOM Proposition 2.2 and case i. of SOM Lemma 2.3 apply, while for \( N \geq 11 \), part ii. and case iii. apply, respectively. For \( N \leq 10 \), one shareholder plays strategy Rubber-stamping and \( N - 1 \) shareholders play strategy CAIS. For \( N \geq 11 \), again one shareholder plays strategy Rubber-stamping and four shareholders play strategy CAIS, while \( N - 5 \) play strategy UNIS. The consequences for decision quality are reported in Table 2.1. The example illustrates the main insight: Decision quality is weakly higher with a PA than without.

While SOM Example 2.1 is restricted to decision qualities \( q_B = 0.8, \) and \( q_P = 0.7 \), we now vary them in the next numerical example, which is illustrated in SOM Figure 2.3. The upper left panel depicts the benchmark scenario that no proxy advisor is admitted, based on SOM Proposition 2.1. The upper right panel depicts decision quality in the game with a PA, based on SOM Lemma 2.3. The lower panel depicts the difference in decision quality, i.e., the effect of admitting a PA. There are several observations to make.

\(^{66}\)SOM Example 2.1 differs from Example 1 in that the board is better informed: \( q_B = 0.8, \) instead of 0.75. As a consequence, the symmetric strategy profile CAIS is no longer an equilibrium, as \( \ell_P \leq \ell_B - \ell_S \) or \( \lfloor (\ell_B - \ell_P)/\ell_S \rfloor \geq 1 \).
First, the difference is never negative, reflecting Proposition 3 in the main text that decision quality weakly improves by the introduction of a PA. Second, decision quality changes smoothly with the signal qualities of the board $q_B$ and of the PA $q_P$. Notice in particular, that the smooth profile in the right upper panel of SOM Figure 2.3 is based on the six cases of SOM Lemma 2.3, corresponding to the six regions in SOM Figure 2.1. Indeed, decision quality changes continuously at every boundary between these six cases of SOM Lemma 2.3, as it can be generally checked when plugging in the condition for the boundary into the two expressions for decision quality for the corresponding cases. Third, as the figure suggests, decision quality turns out to be always weakly increasing in $\ell_P$ and $\ell_B$. This is a notable difference to the analysis of symmetric equilibria, where non-monotonicity and discontinuities arise. For symmetric equilibria, a PA weakly improved decision quality under the Assumption BIB. Relaxing it led to the possibility that a PA worsens decision quality and also that increasing the quality of the PA worsens decision quality. Now, for asymmetric equilibria, decision quality is always weakly improved by the presence of a PA and it is monotonically and smoothly increasing in the quality of the PA. This also holds for the quality of the board.

Finally, considering that $\ell_P$ close to zero is similar to having no PA at all and that decision quality is weakly increasing in $\ell_P$, it is intuitive that a PA weakly improves decision quality. We analytically prove this main result (Proposition 3 in the main text) in Appendix A. Overall, the analysis of asymmetric equilibria may differ from the analysis of symmetric equilibria when it comes to some comparative statics, but it supports the main results. First, the shareholders’ incentive to invest in own research is fostered by the presence of a PA, with maximal research incentives when the PA’s information quality is similar to the board’s. Second, decision quality weakly improves due to the presence of a PA.

### Table 2.1: Decision quality in Pareto-efficient asymmetric equilibria. Illustration of SOM Propositions 2.1 and 2.2 and SOM Lemma 2.3 for $q_B = 0.8$, $q_P = 0.7$, and $q_S = 0.6$, i.e., SOM Example 2.1.

<table>
<thead>
<tr>
<th>Setting</th>
<th>Decision quality</th>
<th>$N = 3$</th>
<th>$N = 5$</th>
<th>$N = 21$</th>
<th>$N = 101$</th>
<th>$N = 1,001$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No PA</td>
<td>$\Pi^{no-PA}$</td>
<td>0.8</td>
<td>0.8</td>
<td>0.867</td>
<td>0.983</td>
<td>1</td>
</tr>
<tr>
<td>With PA</td>
<td>$\Pi(\sigma^*)$</td>
<td>0.812</td>
<td>0.824</td>
<td>0.884</td>
<td>0.984</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 2.3: Decision quality in the Pareto-efficient strategy profile in the parameter space for \( q_S = 0.6 \) and \( N = 15 \). The first panel depicts decision quality when there is no PA based on SOM Prop. 2.1. The second panel depicts decision quality when there is a PA based on SOM Prop. 2.2 and SOM Lemma 2.3. The third panel depicts the difference in decision quality, i.e., the improvement in decision quality due to the PA.